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Black hole thermodynamics in Scalar-Tensor-Vector-Gravity model

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Abstract. The thermodynamic properties of the Schwarzschild–MOG and Kerr–MOG black holes within the Scalar–Tensor–Vector Gravity (STVG) framework are revisited with a consistent treatment of the vector potential in the first law. We show that deriving the entropy from both the area law and the first law yields the same result once STVG contributions are handled properly. Closed expressions for the Hawking temperature, entropy, free energy, and heat capacity are presented in terms of the black hole parameters, clarifying the role of the coupling α. We also outline the rotating case and comment on lifetime estimates. The analysis consolidates and streamlines prior results and resolves methodological ambiguities in earlier treatments.

Key words: Hawking radiation, Thermodynamic properties, STVG, Schwarzchild-MOG, entropy.

I. INTRODUCTION

Hawking's discovery that black holes radiate thermally established a deep connection between gravity, quantum field theory, and statistical mechanics, assigning to black holes a temperature proportional to surface gravity and an entropy proportional to horizon area [1–4]. Within general relativity (GR), these results cohere elegantly with the four laws of black hole mechanics and have been probed across a wide range of classical spacetimes.

At the same time, astrophysical and cosmological data motivate testing gravity beyond GR. Scalar–Tensor–Vector Gravity (STVG, also called MOG) augments the metric with a running effective gravitational coupling and a massive vector field, modifying dynamics on galactic and strong–gravity scales while keeping the theory covariant [5, 6]. In black hole spacetimes of STVG (often dubbed Schwarzschild–MOG and Kerr–MOG), the geometry formally resembles charged GR metrics, but the "charge" is sourced by the additional vector degree of freedom rather than electromagnetism [6]. This distinction is crucial in thermodynamics, where the vector potential contributes an extra work term to the first law.

A recurring issue in the literature is that thermodynamic quantities can appear inconsistent if the vector contribution is omitted or treated analogously to the electromagnetic case without care. For instance, different routes to the entropy (area law vs. integrating the first law) must agree when all STVG terms are accounted for. Likewise, the free energy and heat capacity should reduce smoothly to the Schwarzschild results in the GR limit $\alpha \to 0$ and display a controlled α-dependence away from it. Recent work has emphasized both quantum/statistical corrections (e.g., logarithmic terms) and phenomenological implications of MOG parameters for evaporation, near-horizon optics, and strong-field observables [9–16].

In this paper we revisit the thermodynamics of Schwarzschild–MOG and Kerr–MOG black holes with three goals: (i) present a compact and self-consistent derivation of T, S, F, and C_V that includes the vector-potential contribution to the first law; (ii) clarify the dependence of these quantities on the coupling α and recover the GR limit; and (iii) indicate how rotation modifies the horizon structure and temperature in Kerr–MOG while preserving the entropy–area relation. Our results bring the different derivations into alignment and provide a convenient baseline for future studies of quantum corrections and observational constraints in STVG [8].

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II. SPACETIME METRIC

In the spherical coordinates (t, r, θ, ϕ), the Schwarzschild-MOG spacetime is given as [6]

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^\theta d\phi^2) \,, \tag{1}$$

which is similar to the Reissner-Nordström metric, where G is a modified gravitational constant related to Newtonian gravitational constant G_N as fallows $G = G_N(1 + \alpha)$, quantity $Q = \sqrt{\alpha G_N} M$ is the coupling constant regarded to the external potential in STVG which is given as [6]

$$\Phi_\mu = \Phi_t(1, 0, 0, 0) \,, \qquad \Phi_t = -\frac{Q}{r} = -\frac{\sqrt{\alpha G_N} M}{r} \,, \tag{2}$$

and α is positively defined MOG parameter. The radius of the outer and inner horizons of Schwarzschild-MOG black hole is located at $r_\pm = G_N M(1+\alpha\pm\sqrt{1+\alpha})$. Notice that the outer horizon is greater than the Schwarzschild radius, i.e. $r_+ \geq 2G_N M$ and dependence of the horizon from MOG parameter α is illustrated in Fig. 1.

III. THERMODYNAMICS OF SCHWARZSCHILD-MOG BLACK

Certainly, studying the thermodynamic properties of black holes in Scalar-Tensor-Vector-Gravity (STVG) and investigating the influence of the MOG parameter on these quantities is an intriguing endeavor. We begin by considering the first law of black hole thermodynamics in STVG, which can be expressed as:

$$dM = TdS + \Phi_t dQ \,, \tag{3}$$

where T and S denote the Hawking temperature and entropy of the black hole, respectively. While the second term of equation (3) emerges due to the presence of STVG, and it has the potential to impact the thermodynamic properties of the black hole in this particular modified theory.

The Hawking temperature on the surface of the Schwarzschild-MOG black hole can be determined as follows:

$$T = \frac{1}{4\pi}\left(\frac{dg_{tt}}{dr}\right)\bigg|_{r=r_+} = \frac{1}{2\pi G_N M\sqrt{1+\alpha}\left(1+\sqrt{1+\alpha}\right)^2} \,, \tag{4}$$

Remarkably, this expression aligns with the findings reported in Ref. [7]. As one can see from equation (4) that the Hawking temperature of the black hole in STVG decreases due to the influence of MOG parameter in the black hole's spacetime, as illustrated in Fig. 1. It is well-know that in the Schwarzschild spacetime the Hawking temperature of the black hole is determined as $T_0 = 1/(8\pi G_N M)$, which can be reduced by setting $\alpha = 0$ in the expression (4). Since the MOG parameter is positively defined, the temperature of the Schwarzschild-MOG black hole is small than that is predicted in the Schwarzschild spacetime for non-zero value of the MOG parameter.

Another important quantity in black hole thermodynamics is the entropy of the black hole. Using the area law the entropy of the Schwarzschild-MOG black hole is determined as [7]

$$S_A = \frac{A}{4G} = \frac{\pi r_+^2}{G_N(1+\alpha)} = \pi G_N M^2\left(1+\sqrt{1+\alpha}\right)^2 \,, \tag{5}$$

on other hand, using the first law of the black hole thermodynamics in (3), the same expression for the entropy of the black hole can be derived as

$$\begin{aligned} S_T &= \int \frac{dM}{T} - \int \frac{\Phi_t dQ}{T} = \int \frac{dM}{T}\left(1 - \frac{\alpha G_N M}{r_+}\right) \\ &= \int \frac{dM}{T}\frac{1}{\sqrt{1+\alpha}} = \pi G_N M^2\left(1+\sqrt{1+\alpha}\right)^2 \,, \end{aligned} \tag{6}$$

which coincides with the result in (5), and one can obtain $S_A = S_T = S$. However, in the paper [7], the result for S_T is given as $S_T = \pi G_N M^2\left(1+\alpha+\sqrt{1+\alpha}\right)\left(1+\sqrt{1+\alpha}\right)$ which is different due to the extra factor $\sqrt{1+\alpha}$. Here we can conclude that the entropy of the black hole determined via using area law and the first law of black hole thermodynamics give the same result. In the Schwarzschild spacetime the entropy is determined as $S_0 = 4\pi G_N M^2$. It turns our that the entropy of

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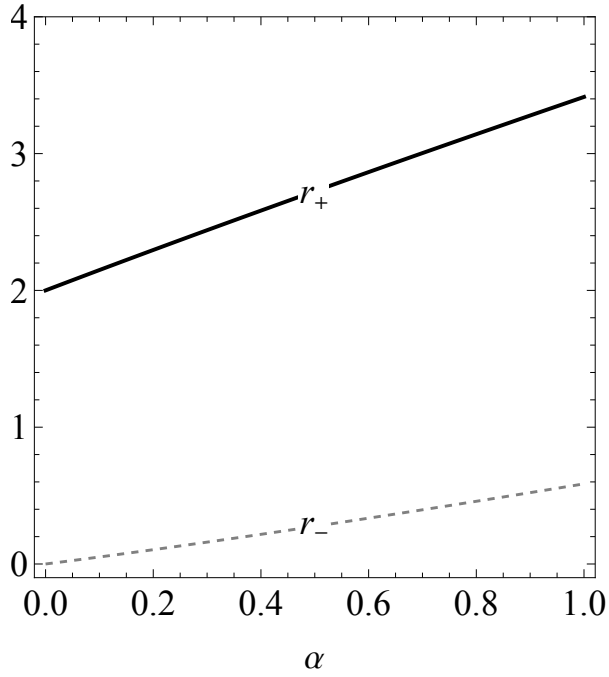


FIG. 1. Dependence the horizon of the Schwarzschild-MOG black hole form MOG parameter α.

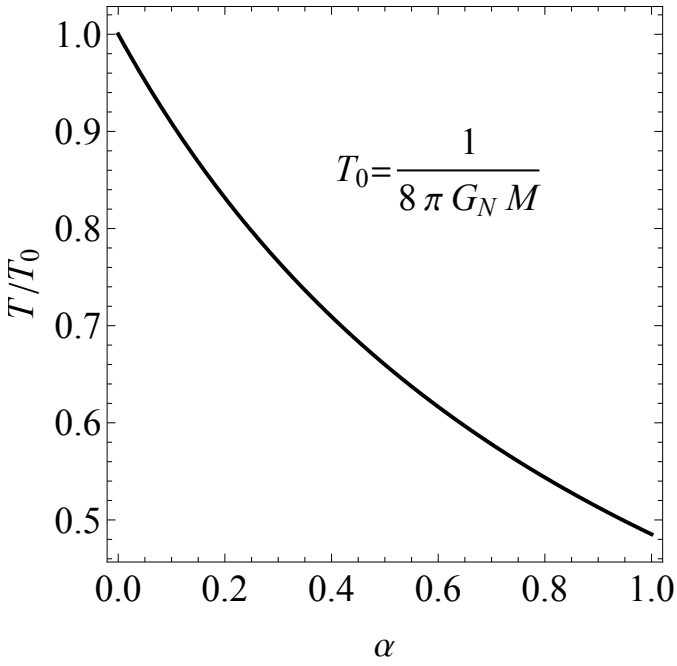


FIG. 2. Dependence of the Hawking temperature of the Schwarzschild-MOG black hole form MOG parameter α.

the Schwarzschild-MOG black hole S is always greater that entropy of the Schwarzschild black hole S_0 for for non-zero value of the MOG parameter which is shown in Fig.3.

Now, utilizing the expressions for the temperature and entropy, we can determine the free energy of the Schwarzschild-MOG black hole as presented in [17]:

$$F = M - ST - Q\Phi_t = \left(2 - \frac{3}{2\sqrt{1+\alpha}}\right) M \,. \tag{7}$$

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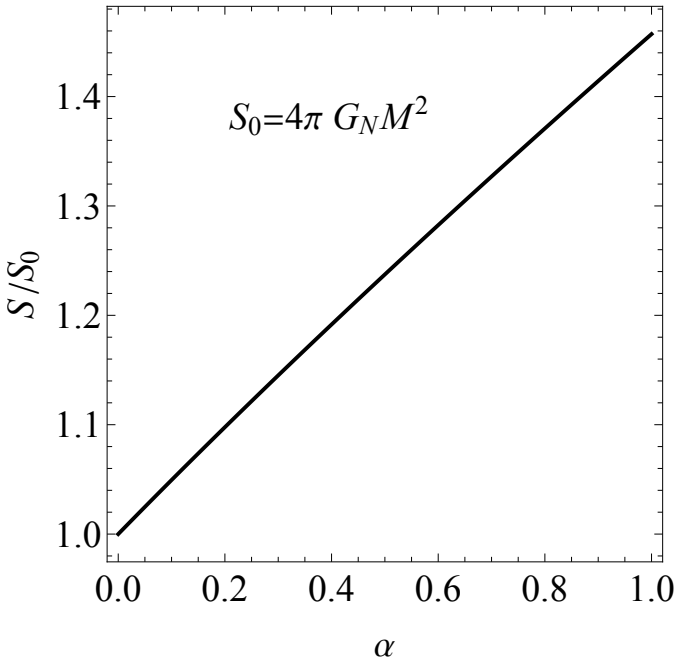


FIG. 3. Dependence of the entropy of the Schwarzschild-MOG black hole from MOG parameter α.

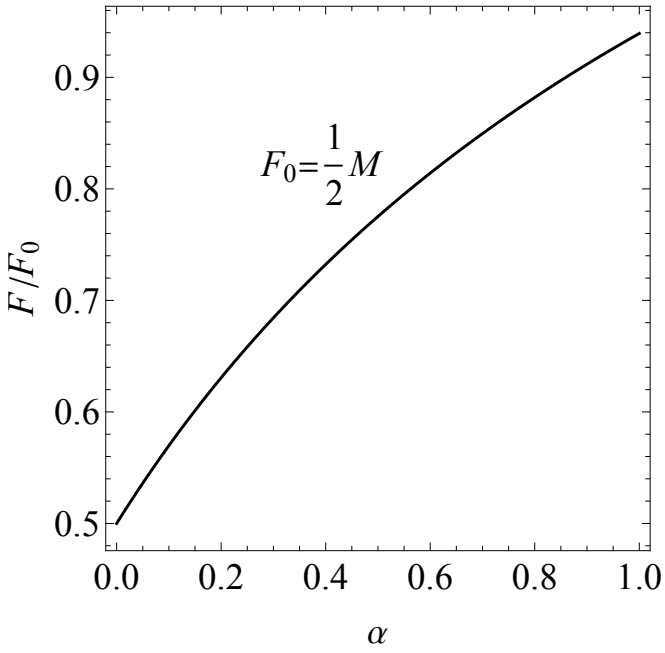


FIG. 4. Dependence of the free energy of the Schwarzschild-MOG black hole from MOG parameter α.

Again the last term arises due to external field occurring in this particular modified gravity. In the Schwarzschild spacetime it reduces to $F_0 = M/2$ which ca be derived from the above equation. In Figure 4, we provide dependence of the free energy of the black hole from the MOG parameter. It is important to note that this expression for the free energy is distinct and differs from the formulations derived in two different ways in the paper [7] due to the variations in the expressions for the entropy, denoted as S_A and S_T. The results for the entropy and free energy of the black hole as presented in the paper [7] are depicted comparison with our findings. In our further calculations, we simply define the entropy as S without any sub-indices, or $S = S_A = S_T$.

Now we focus on calculating the heat capacity of the Schwarzschild-MOG black hole. From the thermodynamics course the isochoric heat capacity is given as $C_V = T(\partial S/\partial T)$, which depends on the entropy of the black hole. Using this expression

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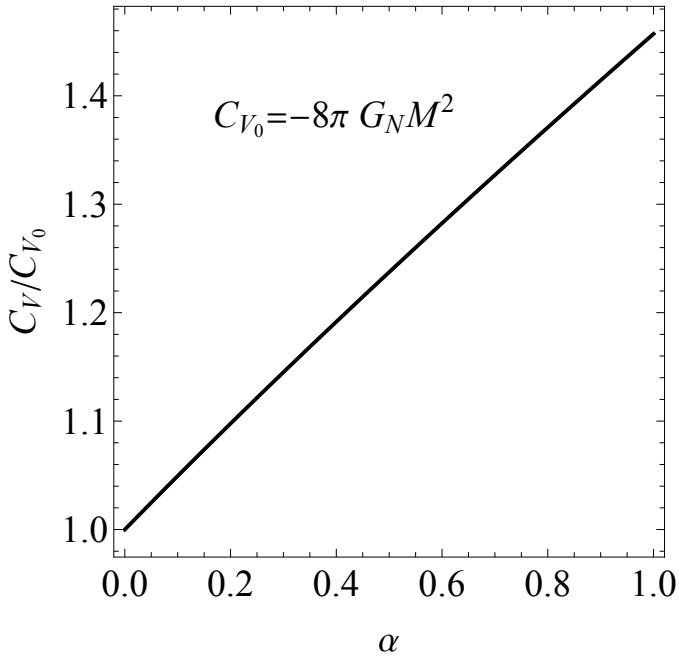


FIG. 5. Dependence of the heat capacity of the Schwarzschild-MOG black hole from MOG parameter α.

heat capacity of the Schwarzschild-MOG black hole can be determined as

$$C_V = T\frac{\partial S}{\partial M}\left(\frac{\partial T}{\partial M}\right)^{-1} = -2\pi G_N M^2\left(1+\sqrt{1+\alpha}\right)^2 < 0\,, \tag{8}$$

which disagrees with the result in [7]. It is easy to see that the isochoric heat capacity of the Schwarzschild-MOG black hole is lass than that is predicted in general relativity. However, it's absolute value greater than that is in the Schwarzschild spacetime. Therefore fractional heat capacity of the black hole in STVG increases with increasing of the MOG parameter which is shown in Fig.5.

Here, we will examine the corrections to the thermodynamic quantities of the Schwarzschild-MOG black hole. In particular, we derive the expected logarithmic correction term to the black hole entropy with proper way. Hereafter introducing new parameter β as the inverse of temperature $\beta = 1/T$. Finally, by utilizing the expressions for the temperature and entropy of the Schwarzschild-MOG black hole, the correction term to the entropy assumes the following form:

$$\begin{aligned}\mathcal{S} &= S - \frac{1}{2}\ln(ST^2)\\ &= \pi G_N M^2\left(1+\sqrt{1+\alpha}\right)^2 + \frac{1}{2}\ln\left[4\pi(1+\alpha)\left(1+\sqrt{1+\alpha}\right)^2\right]\,.\end{aligned} \tag{9}$$

Notice that the dimension of the second term of the entropy in (9) is not correct however this issue can be solved by restoring fundamental constants, such as speed of light, Plank constant and Boltzmann constant.

IV. THERMODYNAMICS OF KERR-MOG BLACK HOLE

Here we briefly discuss the thermodynamics properties of a rotating black hole described by the Kerr-MOG black hole spacetime [6]:

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)^2 d\phi - a dt\right]^2\,, \tag{10}$$

where $\Delta = r^2 - 2GMr + GQ^2 + a^2$ and $\Sigma = r^2 + a^2\cos^2\theta$, a is a spin parameter along the mass parameter M and MOG parameter α. One needs to keep in mind that the external four-potential is given as

$$\Phi_\mu = \frac{Qr}{\Sigma}\left(-1, 0, 0, a\sin^2\theta\right)\,. \tag{11}$$

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The outer and inner horizons of Kerr-MOG black hole in STVG are located at $r_\pm = G_N M\left(1+\alpha \pm \sqrt{1+\alpha-a_*^2}\right)$, where $a_* = a/(G_N M)$, and Hawking temperature of Kerr-MOG black hole takes a form:

$$T = \frac{1}{4\pi G_N M(1+\alpha)} \frac{\sqrt{1+\alpha-a_*^2}}{1+\frac{\alpha}{2}+\sqrt{1+\alpha-a_*^2}}\,, \tag{12}$$

which vanishes in the case of the extreme Kerr-MOG black hole when $a_* = \sqrt{1+\alpha}$, while in the case when $a_* = 0$ the expression (12) reduces to (4). Using the area law the entropy of Kerr-MOG black hole reads

$$S = \frac{A}{4G} = 2\pi G_N M^2\left(1+\frac{\alpha}{2}+\sqrt{1+\alpha-a_*^2}\right)\,, \tag{13}$$

which also can be derived from the first law of black hole thermodynamics $dM = TdS + \Omega dJ + \Phi_t dQ$. Once the Hawking temperature and entropy of black hole are derived, the expression for other thermodynamic quantities, namely, heat capacity, free energy as well as lifetime of the Kerr-MOG black hole can immediately be derived. However, one has to include the effect of black hole rotation in each thermodynamics quantities in proper way.

V. CONCLUSION

We provided a compact, consistent derivation of thermodynamic quantities for Schwarzschild–MOG and Kerr–MOG black holes in STVG. For Schwarzschild–MOG, the temperature (4) decreases with α, while the entropy from the area law and the first law agrees (5)–(6). The free energy (7) and the negative heat capacity (8) display a characteristic α-dependence, and standard logarithmic corrections (9) apply once fundamental constants are restored. In Kerr–MOG, rotation combines with α to control the horizon structure and T via (12). These results align the different derivations in the literature and offer a clear baseline for future quantum corrections and observational constraints in STVG.

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