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Distinguishable feature of electric and magnetic charges of black hole

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Abstract. From a geometrical perspective, it is clear that distinguishing between electrically and magnetically charged Reissner-Nordström black holes is not feasible. One approach to highlighting the differences between these solutions is to analyze the motion of charged test particles in the vicinity of a charged black hole and examine how the charge coupling parameters (σ_e, σ_m) affect the stability of circular orbits. Additionally, we can investigate the fundamental frequencies—such as Keplerian, Larmor, and epicyclic frequencies—of charged particles orbiting around a charged black hole. Furthermore, we explore the synchrotron radiation emitted by charged particles accelerated by a charged black hole and estimate the intensity of radiation from relativistic charged particles. Finally, we discuss how our findings could potentially be applied to the study of relativistic jets and magneto-hydrodynamical outflows.

Key words: Reissner-Nordström black hole, fundamental frequencies, synchrotron radiation, test particles and massless photons

I. INTRODUCTION

According to the classical theory of gravity black holes are rather simple among gravitational compact objects, characterized by three main parameters, namely its mass, angular momentum, and charge. It is well-known that a black hole with such parameters can be a good candidate for the astrophysical black hole, however, its charge is negligibly small in astrophysical situations. The spacetime of such a black hole can be described by the well-known Kerr-Newman metric which is a generalized form of the Kerr metric [1, 2]. Basic idea about the theory of the electromagnetic field can be found in the textbook in electrodynamics and electrostatics [3–15].

Recent observations providing evidence for astrophysical black holes, such as images of supermassive black holes [16–21], X-ray flares attributed to orbital motion of matter near these black holes [22–24], and other stellar X-ray sources, have generated new motivations for investigating the dynamical motion of test particles and massless photons within the framework of general relativity and alternative theories of gravity. While the motion of neutral particles is governed solely by the geodesic equations, which directly probe the geodesic structure of spacetime, charged test particles are subject to both gravitational and electromagnetic forces. This interaction opens up the possibility of examining the electromagnetic properties of black holes, contributing to their complex observational phenomenology in high-energy electromagnetic wavelengths.

It is also widely accepted that high-energy cosmic rays are produced through the acceleration of relativistic charged particles in the presence of external electromagnetic fields near black holes [25–30]. The high-energy acceleration of ionized particles via the electric Penrose process has been explored in [31]. Additionally, the potential for high-energy collisions between particles in the vicinity of regular black holes has been studied in Refs. [32–35]. These studies highlight the role of black holes in the production and acceleration of high-energy particles, contributing significantly to the understanding of extreme astrophysical environments and phenomena.

In the Boyer-Lindquist coordinates $x^\alpha = (t, r, \theta, \phi)$, a rotating charged black hole with mass of M, spin of a, and charge of

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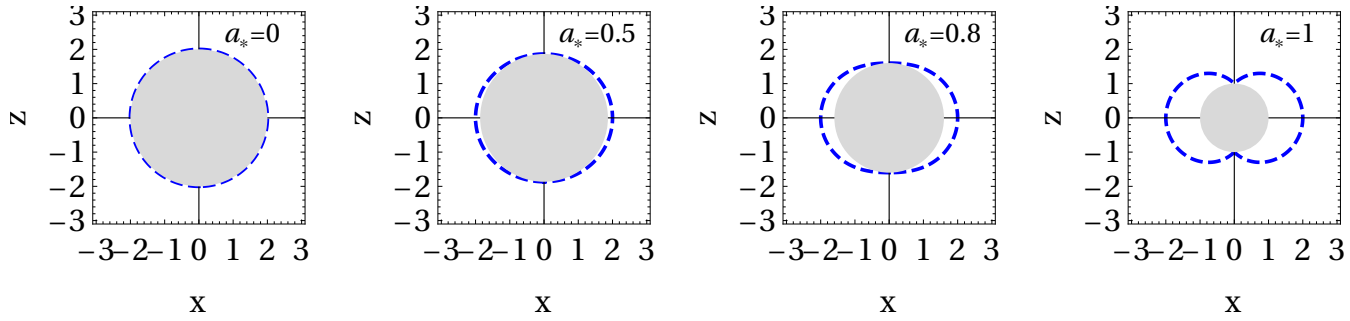


FIG. 1. Dependence of ergosphere from the black hole spin.

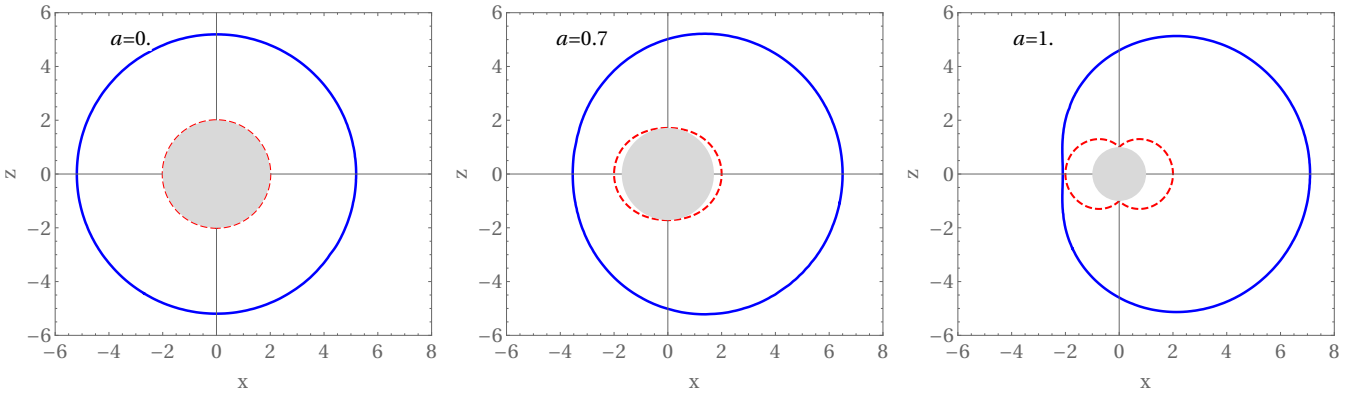


FIG. 2. Dependence of ergosphere from the black hole spin.

Q, is described by Kerr-Newman metric as [36]

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)d\phi - adt\right]^2 , \tag{1}$$

where $\Sigma = r^2 + a^2\sin^2\theta$, $\Sigma = r^2 + a^2\sin^2\theta$, $\Delta = r^2 - 2Mr + a^2 + Q^2$. The radius of the outer horizon can found as larger solution equation $\Delta = 0$, or $r_h = M + \sqrt{M^2 - a^2 - Q^2}$ which is smaller then the Schwarzschild radius, i.e., $r_h \leq 2M$. On the other hand, the radius of ergosphere can be determined as $r_{\rm erg} = M + \sqrt{M^2 - a^2\cos^2\theta - Q^2}$ which is slightly larger the black hole horizon depending on the black hole's parameters as shown in Fig. 1.

As mentioned in the Paper I that Q can be either electric charge ($Q = Q_e$) or magnetic charge ($Q = Q_m$) of the black hole and can not be exist simultaneously. The corresponding components of the associated electromagnetic vector potential:

$$A_\mu = \frac{rQ_e}{\Sigma}\left(-1, 0, 0, a\sin^2\theta\right) , \qquad \text{ECBH} , \tag{2}$$

$$A_\mu = \frac{Q_m\cos\theta}{\Sigma}(-a, 0, 0, r^2 + a^2) , \qquad \text{MCBH} . \tag{3}$$

II. CHARGED PARTICLE DYNAMICS

We study dynamical motion of charged particle around charged black hole. The Lagrangian for charged particle with mass of m and charge q can be expressed as

$$\mathscr{L} = \frac{1}{2}mg_{\alpha\beta}U^\alpha U^\beta + qU^\alpha A_\alpha , \tag{4}$$

where $U^\alpha = dx^\alpha/d\lambda$ is the four-velocity of the charged particle, λ an affine parameter. From the Lagrangian (4), one can derive equation of motion for charged particle as

$$\frac{dU^\alpha}{d\lambda} + \Gamma^\alpha_{\mu\nu} U^\mu U^\nu = \frac{q}{m} F^\alpha_{\ \beta} U^\beta \ . \tag{5}$$

Using the properties of stationary and asymmetric of the Lagrangian (4), conserved quantities, namely, the energy E and angular momentum L, can be determined as

$$g_{tt}U^t + g_{t\phi}U^\phi + \frac{q}{m}A_t = -\frac{E}{m} = -\mathcal{E} = \text{const} \ , \tag{6}$$

$$g_{t\phi}U^t + g_{\phi\phi}U^\phi + \frac{q}{m}A_\phi = \frac{L}{m} = \mathcal{L} = \text{const} \ , \tag{7}$$

which allow to find U^t and U^ϕ as

$$U^t = -\frac{(\mathcal{L} - \frac{q}{m}A_\phi)g_{t\phi} + (\mathcal{E} + \frac{q}{m}A_t)g_{\phi\phi}}{g_{tt}g_{\phi\phi} - g^2_{t\phi}} \ , \tag{8}$$

$$U^\phi = \frac{(\mathcal{L} - \frac{q}{m}A_\phi)g_{tt} + (\mathcal{E} + \frac{q}{m}A_t)g_{t\phi}}{g_{tt}g_{\phi\phi} - g^2_{t\phi}} \ , \tag{9}$$

where $\mathcal{E}$ and $\mathcal{L}$ are, respectively, the specific energy and specific angular momentum of charged particle measured at infinity. Using the normalization of the four-velocity $U_\alpha U^\alpha = -1$, one can write the following equation []:

$$g_{rr}(U^r)^2 + g_{\theta\theta}(U^\theta)^2 + V(r,\theta) = 0 \ , \tag{10}$$

where

$$V(r,\theta) = \frac{1}{g_{tt}g_{\phi\phi} - g^2_{t\phi}} \left[2g_{t\phi}\left(\mathcal{E} + \frac{q}{m}A_t\right)\left(\mathcal{L} - \frac{q}{m}A_\phi\right) \right.$$
$$\left. + g_{tt}\left(\mathcal{L} - \frac{q}{m}A_t\right)^2 + g_{\phi\phi}\left(\mathcal{E} + \frac{q}{m}A_t\right)^2 \right] + 1 \ . \tag{11}$$

It is convenience to consider motion in equatorial plane, since most astrophysical situation... In this case four-velocity of particle can be written as $U^\alpha = (U^t, 0, 0, U^\phi)$, again using the normalization of four-velocity, after performing simple algebraic manipulations, one can obtain

$$U^t = \frac{1}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}} \ , \tag{12}$$

$$\mathcal{E} = -\frac{g_{tt} + \Omega g_{t\phi}}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}} - \frac{q}{m}A_t \ , \tag{13}$$

$$\mathcal{L} = \frac{g_{t\phi} + \Omega g_{\phi\phi}}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}} + \frac{q}{m}A_\phi \ , \tag{14}$$

where $\Omega = U^\phi/U^t = d\phi/dt$ is an angular velocity or Keplerian frequency.

III. FUNDAMENTAL FREQUENCY OF CHARGED PARTICLE

Keplerian frequency: Now we concentrate on determining of Keplerian frequency of the charged particle orbiting around Kerr-Newman black hole. From equation (34) one can see that the radial and vertical frequencies $(\Omega_r, \Omega_\theta)$ depend on the the background geometry, the external electromagnetic field and also parameters of the test particle. Once background spacetime geometry and external magnetic field are given then one can immediately determine Ω_r and Ω_θ, however keep in mind that they st

Now we consider motion in circular orbit with the following form four-velocity, $U^\alpha = U^t(1, 0, 0, \Omega)$, where time component of velocity, U^t, conserved quantities, i.e., $\mathcal{E}$, $\mathcal{L}$, satisfy the expressions (12)-(14). As we know that time and azimuthal components of equation (5) represents conserved quantities. However, after performing simple algebraic manipulations in remaining

cases, (i.e. $\alpha = r, \theta$), equations for radial and vertical motions for the charged particle can be expressed as

$$g_{tt,r} + 2\Omega g_{t\phi,r} + \Omega^2 g_{\phi\phi,r} = -\frac{2q}{mU^t}\left(F_{rt} + \Omega F_{r\phi}\right) \,, \tag{15}$$

$$g_{tt,\theta} + 2\Omega g_{t\phi,\theta} + \Omega^2 g_{\phi\phi,\theta} = -\frac{2q}{mU^t}\left(F_{\theta t} + \Omega F_{\theta\phi}\right) \,. \tag{16}$$

Note that the physical meaning of quantity Ω in above equations are different. The solution of Eq.(15) represents Keplerian frequency of charged particle, while solution of Eq.(16) corresponds to Gyroscopic frequency which depend on parameters of charged particle e, m, as well as the black hole's parameters and the external electromagnetic fields. For neutral particle, i.e. $q = 0$, the explicit expressions for Keplerian and Gyroscopic frequencies in Kerr-Newman spacetime can be easily found as

$$\Omega_K = \frac{1}{\frac{r^2}{\sqrt{Mr-Q^2}} \pm a} \,, \tag{17}$$

$$\Omega_G = \frac{\pm a}{r^2 + a^2 \pm \frac{ir^2\sqrt{\Delta}}{\sqrt{2Mr-Q^2}}} \,, \tag{18}$$

here one can see that Gyroscopic frequency is complex function, and imaginary part is responsible for damping of the oscillation.

As one can see from equations (15) and (16) that structure of these equations are equivalent. It is enough to solve one of these equations. In order to find, Ω, in equation (15) one needs to solve the following fourth order equation:

$$\Omega^4 + \mathscr{A}\Omega^3 + \mathscr{B}\Omega^2 + \mathscr{C}\Omega + \mathscr{D} = 0 \,, \tag{19}$$

where coefficients $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$, and $\mathscr{D}$ are defined as

$$\mathscr{A} = \frac{4}{\eta}\left[\frac{g_{t\phi,r}}{g_{\phi\phi,r}} + 2\left(\frac{qA_{t,r}}{mg_{\phi\phi,r}}\right)^2\left(g_{\phi\phi} + g_{t\phi}\frac{A_{\phi,r}}{A_{t,r}}\right)\right] \,, \tag{20}$$

$$\begin{aligned}\mathscr{B} = \frac{2}{\eta}\Bigg[&\frac{g_{tt,r}}{g_{\phi\phi,r}} + 2\left(\frac{g_{t\phi,r}}{g_{\phi\phi,r}}\right)^2 + 8g_{t\phi}\frac{q^2A_{t,r}A_{\phi,r}}{(mg_{\phi\phi,r})^2} \\ &+2g_{\phi\phi}\left(\frac{qA_{t,r}}{mg_{\phi\phi,r}}\right)^2 + 2g_{tt}\left(\frac{qA_{\phi,r}}{mg_{\phi\phi,r}}\right)^2\Bigg] \,,\end{aligned} \tag{21}$$

$$\mathscr{C} = \frac{4}{\eta}\left[\frac{g_{tt,r}g_{t\phi,r}}{(g_{\phi\phi,r})^2} + 2\left(\frac{qA_{t,r}}{mg_{\phi\phi,r}}\right)^2\left(g_{t\phi} + g_{tt}\frac{A_{\phi,r}}{A_{t,r}}\right)\right] \,, \tag{22}$$

$$\mathscr{D} = \frac{1}{\eta}\left[\left(\frac{g_{tt,r}}{g_{\phi\phi,r}}\right)^2 + 4g_{tt}\left(\frac{qA_{t,r}}{mg_{\phi\phi,r}}\right)^2\right] \,, \tag{23}$$

$$\eta = 1 + 4g_{\phi\phi}\left(\frac{qA_{\phi,r}}{mg_{\phi\phi,r}}\right)^2 \,. \tag{24}$$

Before solving the polynomial equation (19), we introduce new variable, $x = \Omega - \mathscr{A}/3$, then one can have

$$x^4 + px^2 + qx + u = 0 \,, \tag{25}$$

with

$$p = \mathscr{B} - \frac{3}{8}\mathscr{A}^2 \,, \quad q = \mathscr{C} + \frac{1}{8}\mathscr{A}\left(\mathscr{A}^2 - 4\mathscr{B}\right) \,,$$
$$u = \mathscr{D} - \frac{1}{256}\mathscr{A}\left(3\mathscr{A}^3 - 16\mathscr{A}\mathscr{B} + 64\mathscr{C}\right) \,.$$

Hereafter performing algebraic manipulations, equation (25) can be expressed as

$$\begin{aligned}&\left(x^2 + \sqrt{s-p}x + \frac{s}{2} - \frac{q}{2\sqrt{s-p}}\right) \\ &\times\left(x^2 - \sqrt{s-p}x + \frac{s}{2} + \frac{q}{2\sqrt{s-p}}\right) = 0 \,,\end{aligned} \tag{26}$$

where s is satisfied

$$s^3 - ps^2 - 4us + 4pu - q^2 = 0 \,, \tag{27}$$

and the solutions of the equation (26) can be found as

$$x_{1,2} = -\frac{1}{2}\sqrt{s-p} \pm \frac{1}{2}\sqrt{-s-p+\frac{2q}{\sqrt{s-p}}} \,, \tag{28}$$

$$x_{3,4} = -\frac{1}{2}\sqrt{s-p} \pm \frac{1}{2}\sqrt{-s-p-\frac{2q}{\sqrt{s-p}}} \,. \tag{29}$$

Finally, Keplerian frequency can be obtained as

$$\Omega_i = x_i + \frac{1}{3}\mathcal{A} \,, \quad i = 1-4 \,, \tag{30}$$

Here one needs to find physically meaningful solutions among Ω_i.

Very similar calculation can be done for Gyroscopic frequency in equation (16), in order to find Gyroscopic frequency instead of r one needs to put θ in all equation for Keplerian frequency.

The epicyclic frequencies: It is also interesting to calculate the epicyclic frequencies $(\Omega_r, \Omega_\theta)$ governed by oscillatory motion charged-magnetized particle along radial and vertical direction at stable orbit around black hole in the presence of external magnetic field. Here we will study quasi-periodic oscillation of charged and magnetized particles at given circular orbit. Before move on further that let us expand of the function $V(r,\theta)$ in the form

$$V(r,\theta) \simeq \frac{1}{2}\partial_r^2 V(r,\theta)\delta r^2 + \frac{1}{2}\partial_\theta^2 V(r,\theta)\delta\theta^2 \,. \tag{31}$$

Notice that here we have used the following relations $\partial_r V(r,\theta) = \partial_\theta V(r,\theta) = 0$ which correspond to stationary point of the function $V(r,\theta)$, while the expression, $V(r_0,\theta_0) = 0$, allows to limit of the constants of motion in a circular orbit. Now inserting the expression (11) into (10), one can obtain equation of harmonic oscillatory motion, around the point (r_0,θ_0), for the displacement $\delta_r = r - r_0$, $\delta_\theta = \theta - \theta_0$ in the form:

$$\frac{d^2}{dt^2}\delta_r + \Omega_{\rm r}^2\delta_r = 0, \qquad \frac{d^2}{dt^2}\delta_\theta + \Omega_\theta^2\delta_\theta = 0 \,, \tag{32}$$

where the epicyclic frequencies in (32) can be calculated by

$$\left(\Omega_r^2, \Omega_\theta^2\right) = \frac{1}{2(U^t)^2}\left(\frac{1}{g_{rr}}\partial_r^2 V(r,\theta), \frac{1}{g_{\theta\theta}}\partial_\theta^2 V(r,\theta)\right) \,. \tag{33}$$

Finally, recalling the equations (11)-(14), the explicit form of the epicyclic frequencies $(\Omega_r, \Omega_\theta)$ for the charged particle orbiting around black hole can be expressed as

$$\begin{aligned}
g_{ii}\Omega_i^2 = {} & \frac{1}{2}\left(g_{tt,ii} + 2\Omega g_{t\phi,i} + \Omega^2 g_{\phi\phi,ii}\right) + 2g^{t\phi}(g_{tt,i} + \Omega g_{t\phi,i})(g_{t\phi,i} + \Omega g_{\phi\phi,i}) \\
& + g^{tt}(g_{tt,i} + \Omega g_{t\phi,i})^2 + g^{\phi\phi}(g_{t\phi,i} + \Omega g_{\phi\phi,i})^2 \\
& - \frac{q}{m}\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}\left[A_{t,ii} - 2A_{t,i}\left(g^{tt}(g_{tt,i} + \Omega g_{t\phi,i}) + g^{t\phi}(g_{t\phi,i} + \Omega g_{\phi\phi,i})\right)\right] \\
& - \frac{q}{m}\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}\left[\Omega A_{\phi,ii} - 2A_{\phi,i}\left(g^{t\phi}(g_{tt,i} + \Omega g_{t\phi,i}) + g^{\phi\phi}(g_{t\phi,i} + \Omega g_{\phi\phi,i})\right)\right] \\
& + \left(\frac{q}{m}\right)^2 (g_{tt} + 2\Omega g_{t\phi} + \Omega^2 g_{\phi\phi})\left[g^{tt}(A_{t,i})^2 + 2g^{t\phi}A_{t,i}A_{\phi,i} + g^{\phi\phi}(A_{\phi,i})^2\right] \,, \qquad i = (r,\theta) \,.
\end{aligned} \tag{34}$$

From equation (34) one can see that the radial and vertical frequencies $(\Omega_r, \Omega_\theta)$ depend on the the background geometry, the external electromagnetic field and also parameters of the test particle. Once background spacetime geometry and external magnetic field are given then one can immediately determine Ω_r and Ω_θ, however keep in mind that they still depend Keplerian frequency which is most important in the calculation of the fundamental frequencies.

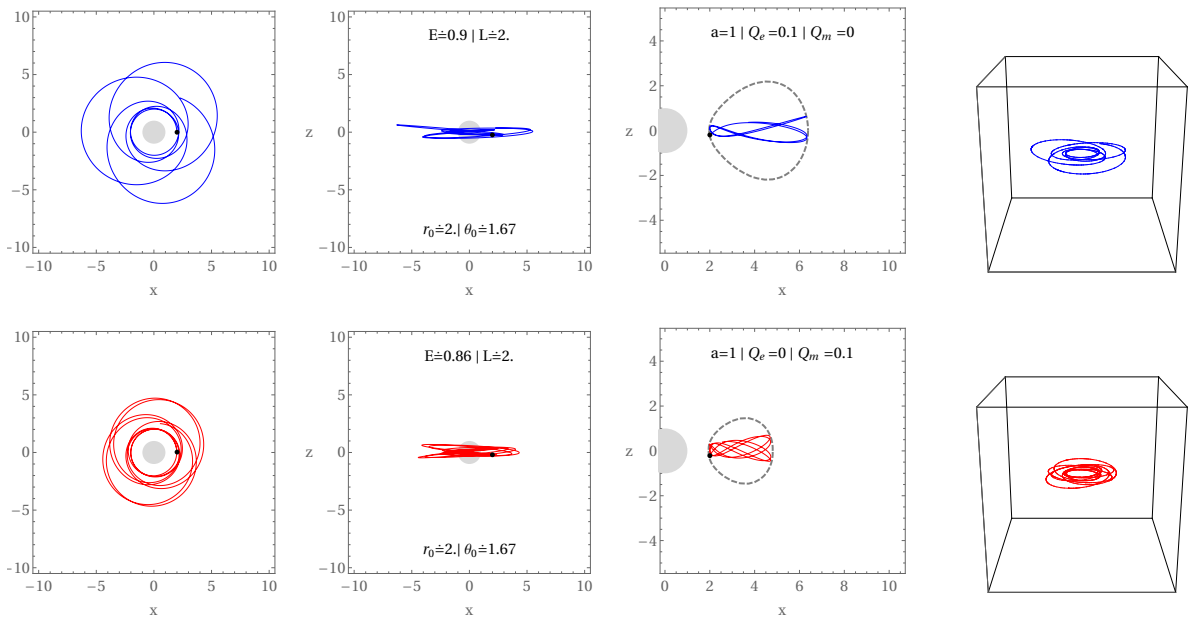


FIG. 3. Charged particle trajectories in the vicinity of electric and magnetic charged Kerr-Newman black hole for the same values of coupling paramters.

IV. SYNCHROTRON RADIATION FROM CHARGED PARTICLE

The acceleration of of charged particle in a curved space can be expressed as, $w^\alpha = U^\beta \nabla_\beta U^\alpha$, on the other hand taking account non-geodesic equation (5), one can write

$$w^\alpha = \frac{q}{m} F^\alpha{}_\beta U^\beta \,, \qquad w_\alpha U^\alpha \equiv 0 \,. \tag{35}$$

The synchrotron radiation spectrum of the relativistic charged particle in curved spacetime can be expressed as [37]

$$I = -\frac{2q^2}{3} w_\alpha w^\alpha = \frac{2q^4}{3m^2} F_{\alpha\gamma} F^{\gamma\beta} U^\alpha U_\beta \,, \tag{36}$$

Again considering circular motion with $U^\alpha = (U^t, 0, 0, U^\phi)$, and taking into account normalization of the four-velocity, hereafter performing simple algebra we obtain the expression for the acceleration the following the form, $w^\alpha = (0, w^r, w^\theta, 0)$, where the components of the acceleration can be defined as

$$w_r = \frac{q}{m} \frac{F_{rt} + \Omega F_{r\phi}}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}} \,, \tag{37}$$

$$w_\theta = \frac{q}{m} \frac{F_{\theta t} + \Omega F_{\theta\phi}}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}} \,, \tag{38}$$

The intensity of the radiating charged particle is

$$I = \frac{2q^4}{3m^2} \frac{1}{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}} \times \left[g^{rr} \left(F_{rt} + \Omega F_{r\phi} \right)^2 + g^{\theta\theta} \left(F_{\theta t} + \Omega F_{\theta\phi} \right)^2 \right] \,, \tag{39}$$

V. PENROSE PROCESS

The Penrose process is a theoretical mechanism proposed by physicist Roger Penrose in 1969, which explains how black holes can lose mass and energy. The process occurs near the event horizon of a rotating (Kerr) black hole and involves the splitting

of a single particle into two parts, one of which falls into the black hole while the other escapes, carrying away energy. This process can result in the black hole losing mass, which is a remarkable feature because black holes are generally thought to only grow by accumulating mass.

One of promising mechanism for energy extraction from black hole is explained by well-known process, so-called Penrose process. The energy efficiency for maximally rotating neutral black hole reaches up to $\eta \sim 21\%$.

A particle near the event horizon of a rotating black hole (specifically, within the ergosphere) can spontaneously decay into two smaller particles. Assume that neutral particle (1) of parameters $(m_1, \mathcal{E}_1, \mathcal{L}_1, \dot{r}_1, \dot{\theta}_1)$ falling on to black hole from infinity and decays into two charged fragments (2) and (3) with parameters $(m_2, \mathcal{E}_2, \mathcal{L}_2, \dot{r}_2, \dot{\theta}_2, q_2)$ and $(m_3, \mathcal{E}_3, \mathcal{L}_3, \dot{r}_3, \dot{\theta}_3, q_3)$ in the ergosphere, where m_i, q_i, $\mathcal{E}_i$, $\mathcal{L}_i$, $\dot{r}_i$ and $\dot{\theta}_i$ are, respectively, the mass, charge, specific energy, specific angular momentum, radial and vertical velocity

The conserved quantities are

$$m_1 = m_2 + m_3 \,, \qquad q_1 = q_2 + q_3 \,, \tag{40}$$

$$m_1\mathcal{E}_1 = m_2\mathcal{E}_2 + m_3\mathcal{E}_3 \,, \quad m_1\mathcal{L}_1 = m_2\mathcal{L}_2 + m_3\mathcal{L}_3 \,, \tag{41}$$

$$m_1\dot{r}_1 = m_2\dot{r}_2 + m_3\dot{r}_3 \,, \quad m_1\dot{\theta}_1 = m_2\dot{\theta}_2 + m_3\dot{\theta}_3 \,, \tag{42}$$

on the other hand particle (1) follows geodesic motion, however, after decaying fragments (2), (3) should follow non-geodesic motion, taking into account the expression for the four-acceleration of particle in (35), one can write

$$0 = w_2^r + w_3^r \,, \qquad 0 = w_2^\theta + w_3^\theta \,, \tag{43}$$

or

$$0 = \frac{q_2}{m_2}\dot{r}_2 + \frac{q_3}{m_3}\dot{r}_3 \,, \qquad 0 = \frac{q_2}{m_2}\dot{\theta}_2 + \frac{q_3}{m_3}\dot{\theta}_3 \,, \tag{44}$$

A related process proposed by Stephen Hawking in 1974, where black holes can emit radiation due to quantum effects near the event horizon, leading to the eventual evaporation of black holes. The Penrose process provides a theoretical foundation for energy extraction methods such as the Penrose engine, where the energy lost by a black hole could be harnessed for practical purposes, such as powering a spacecraft. In summary, the Penrose process is a fascinating and significant phenomenon in theoretical physics, showing how black holes can lose mass and energy through interactions involving charged or neutral particles in their vicinity.

VI. SUMMARY

In this research note, we explore the motion of charged particles around ECBH and MCBH. We analyze how the charge coupling parameter influences the ISCO parameters, including specific energy, specific angular momentum, critical angle, and the innermost stable circular orbit radius. Our numerical results indicate that the ISCO for charged particles around ECBH lies in the equatorial plane and is always farther from the black hole compared to that of neutral particles, with the exact position depending on the charge parameter σ_e. In contrast, for MCBH, the ISCO position can be derived analytically and remains the same as for neutral particles, but is displaced from the equatorial plane by an angle $\theta_0 = \tan^{-1}(2\sqrt{3}/\sigma_m)$, depending on the coupling parameter σ_m. This suggests that charge separation in the accreting matter around MCBH can be observed.

Using the standard approach for deriving fundamental frequencies such as the Keplerian and Larmor frequencies, we explicitly calculate these values for charged particles near a charged black hole. We find that the Keplerian frequency of charged particles can be either higher or lower than that in Schwarzschild spacetime, depending on the sign of the coupling parameter.

We also examine the synchrotron radiation resulting from the acceleration of charged particles near the charged black hole. Explicit expressions for the intensity of radiation from relativistic charged particles in the vicinity of both ECBH and MCBH, due to electromagnetic forces (Coulomb and Lorentz), are derived. Our numerical calculations show that the radiation intensity for accelerated charged particles near MCBH is consistently greater than that around ECBH. Additionally, we find that radially infalling charged particles, with arbitrary radial and angular velocities (u, ω), produce synchrotron radiation with an intensity proportional to $I \sim (2q^4Q^2)/(3m^2r^4)$, though with different factors for each scenario. These results highlight the distinct characteristics of the electric and magnetic charge of the black hole.

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