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Anomaly of the electronic entrophy of underdoped and optimally doped high-Tc cuprates in their superconducting state

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Abstract. This study addresses the anomalous properties of high-temperature superconductors, specifically doped cuprates, which exhibit unique behaviors in their superconducting state compared to conventional metallic (BCS) superconductors. Since the discovery of high-temperature superconductivity in these materials, extensive theoretical and experimental research has been conducted to uncover the mechanisms underlying this phenomenon. However, the exact origins of superconductivity in underdoped, optimally doped, and slightly overdoped cuprates, along with their unusual superconducting properties, remain controversial. Notably, experimental findings indicate that the superconducting transition at the critical temperature $T = T_c$ in high-T_c cuprates resembles the λ-like superfluid transition observed in liquid helium-4. This suggests a unique two-stage superconductivity mechanism: the formation of boson (polaron) Cooper pairs at a temperature $T = T^* > T_c$, followed by the condensation of these pairs into a superfluid Bose liquid at $T = T_c$. In contrast, Fermi-liquid superconductivity, characteristic of BCS theory, is believed to occur only in heavily overdoped cuprates. In this manuscript, the temperature dependence of entropy and heat capacity for the superconducting Bose liquid of polaron Cooper pairs in high-T_c cuprates is analyzed. An analytical expression for the temperature-dependent entropy, $S_e(T)$, is derived for a superfluid 3D Bose liquid in high-T_c cuprates. The results reveal that as temperature decreases below T_c, the entropy $S_e(T)$ sharply drops due to the formation of an energy gap $\Delta_g(T)$ in the excitation spectrum. This behavior closely aligns with experimental observations of the electron entropy in high-T_c cuprates like $YBa_2Cu_3O_{6+x}$.

Key words: High-T_c cuprate superconductivity, BCS-type pseudogap, Fermi-liquid, Entropy of superfluid Bose-liquid.

I. INTRODUCTION

After the discovery of high-temperature superconductivity in doped cuprates, intensive theoretical and experimental studies of the cause of this phenomenon began. Despite such enormous efforts spent on studying the mechanisms of high-temperature superconductivity in underdoped and optimally doped cuprates, the true causes of the origin of superconductivity in them and the anomalous superconducting properties of these high-T_c materials are still controversial. At present, something is already known about the superconducting properties of underdoped, optimally doped and moderately overdoped cuprates, but much is still unknown. Thus, it became known that these high-T_c materials exhibit numerous anomalies in their superconducting properties. Numerous experimental data indicate that the superconducting properties of underdoped, optimally doped and even some overdoped high-T_c cuprates differ greatly from the superconducting properties of conventional metallic superconductors [1-19]. Moreover, the most important and anomalous electronic properties of these high-T_c materials in their superconducting state, which have a direct connection with the mechanism of superconductivity in them, are their thermodynamic properties, which differ significantly from such properties of conventional metallic (i.e. BCS) superconductors. Indeed, according to experimental data on heat capacity [5, 6, 8, 20], the superconducting transition at $T = T_c$ in these unusual superconductors closely resembles the λ-like superfluid transition in He_4 liquid [1, 21]. In this case, the behavior of the electronic heat capacity of underdoped, optimally doped, and slightly overdoped high-T_c cuprates differs significantly from the behavior of the electronic

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heat capacity of BCS superconductors not only at $T = T_c$, but also at low temperatures $T \ll T_c$. In fact, the appearance of unusual high-temperature superconductivity in high-T_c cuprates is a two-stage process [22-24]: the formation of boson (polaron) Cooper pairs at $T = T^* > T_c$ and the subsequent condensation of such boson Cooper pairs into superfluid Bose liquids at $T = T_c$. It should be noted here that the BCS-type Fermi-liquid superconductivity is most likely realized in heavily overdoped cuprates [26]. In contrast, the unusual Bose-liquid superconductivity will be realized in underdoped, optimally doped and slightly overdoped high-T_c cuprates, where polaron Cooper pairs behave as Bose particles [26]. Since the λ-like superconducting transition at $T = T_c$ in these high-T_c materials indicates exactly this. Therefore, in this chapter we will investigate the characteristic temperature dependences of the entropy and heat capacity of the superconducting Bose liquid of polaron Cooper pairs in high-T_c cuprates in their superconducting state (i.e. at $T \ll T_c$ and $T \leq T_c$) [26-30].

II. ENTROPY OF SUPERCONDUCTING BOSE LIQUIDS OF POLARON COOPER PAIRS IN HIGH-T_c CUPRATES.

As mentioned above, Cooper pairing of polaron carriers in underdoped, optimally doped and slightly overdoped cuprates occurs above or much above T_c and results in the formation of strongly bound polaron Cooper pairs that behave as Bose particles. When the temperature is lowered to T_c, the attractive part of the Bose gas of these boson Cooper pairs condenses into a superfluid (superconducting) Bose liquid at $T \leq T_c$ [26]. Therefore, the entropy of such a superfluid Bose liquid is determined from the expression [10]

$$S\left(T\right) = k_B \sum_k \left[\left(n_B\left(k,T\right)+1\right)\ln\left|\left(n_B\left(k,T\right)+1\right)\right| - n_B\left(k,T\right)\ln\left|n_B\left(k,T\right)\right|\right] , \tag{1}$$

where $n_B\left(k,T\right) = [exp(E_B(k)/k_BT - 1]^{-1}$ is the Bose distribution function, $E_B\left(k\right) = \sqrt{(\varepsilon(k)+\widetilde{\mu}_B)^2 - \Delta_B^2(T)}$ is the excitation spectrum of the superfluid Bose liquid, $\varepsilon\left(k\right)$ is the kinetic energy of bosons, $\widetilde{\mu}_B$ is the renormalized chemical potential of interacting bosons, Δ_B is the coherence parameter (or superfluid order parameter) of the attracting bosons (polaron Cooper pairs).

The expression for $E_B\left(k\right)$ can be written as

$$E_B\left(k\right) = \sqrt{\varepsilon^2 + 2\varepsilon\widetilde{\mu}_B\left(T\right) + \Delta_g^2(T)} , \tag{2}$$

where $\Delta_g\left(T\right) = \sqrt{\widetilde{\mu}_B^2\left(T\right) - \Delta_B^2(T)}$, $\varepsilon = \hbar^2k^2/2m_B$, m_B is the mass of bosons.

The expression (1) can be written as

$$S\left(T\right) = k_B \sum_k \left[\frac{\ln\left|\frac{1}{1-\exp(-E_B(k)/k_BT)}\right|}{1-\exp\left(-E_B(k)/k_BT\right)} - \frac{\exp\left(-E_B(k)/k_BT\right)}{1-\exp\left(-E_B(k)/k_BT\right)}\ln\left|\frac{\exp\left(-E_B(k)/k_BT\right)}{1-\exp\left(-E_B(k)/k_BT\right)}\right|\right] , \tag{3}$$

For $\Delta_g\left(T\right) > 0$, the following expressions $[1-\exp\left(-E_B(k)/k_BT\right)]^{-1}$, $\ln\left|\exp\left(-E_B(k)/k_BT\right)\right|$, $\ln\left|1-\exp\left(-E_B(k)/k_BT\right)\right|$ can be expanded into a Taylor series in powers. Then the expression (3) can be written as follows

$$S\left(T\right) \approx k_B \sum_k \left[3\exp\left(-\frac{E_B\left(k\right)}{k_BT}\right) - \exp\left(-\frac{3E_B\left(k\right)}{k_BT}\right)\right] \approx 3k_B \sum_k \exp\left(-\frac{E_B\left(k\right)}{k_BT}\right) . \tag{4}$$

Replacing the summation in the expression (4) with integration, we obtain

$$S\left(T\right) \approx 3\Omega k_B D_B \int_0^\infty \sqrt{\varepsilon}\exp\left(-\frac{E_B\left(k\right)}{k_BT}\right) d\varepsilon, \tag{5}$$

where Ω is the volume of the Bose gas, $D_B = \sqrt{2}m_B^{3/2}/2\pi^2\hbar^3$ is the density of boson states. After replacing the variable $\varepsilon/k_BT = y$, we can write the expression (5) as

$$S\left(T\right) \approx 3\Omega k_B D_B (k_BT)^{3/2} \int_0^\infty \sqrt{y}\exp\left(-\sqrt{y^2 + 2y\widetilde{\mu}_B^*\left(T\right) + \Delta_g^{*2}(T)}\right) dy, \tag{6}$$

where $\widetilde{\mu}_B^*(T) = \widetilde{\mu}_B(T)/k_BT$, $\Delta_g^*(T) = \Delta_g(T)/k_BT$.

Above T_c, $\widetilde{\mu}_B\left(T\right)$ and $\Delta_B\left(T\right)$ become equal to zero and therefore the expression (6) will be

$$S\left(T\right) \approx 3\Omega k_B D_B (k_BT)^{3/2} \int_0^\infty y^{1/2}e^{-y}dy, \tag{7}$$

Below T_c, $\widetilde{\mu}_B\left(T\right) > 0$, $\Delta_B\left(T\right) > 0$ and the entropy will decrease sharply due to the temperature dependence of the exponent in the integrand. The characteristic temperature dependence of the entropy $S(T)$ above T_c and below T_c are shown in Fig. 1. This temperature dependence $S(T)$ is similar to the experimentally observed temperature dependence of the entropy $S(T)$ in high-T_c cuprates, shown in Fig. 2 in [8].

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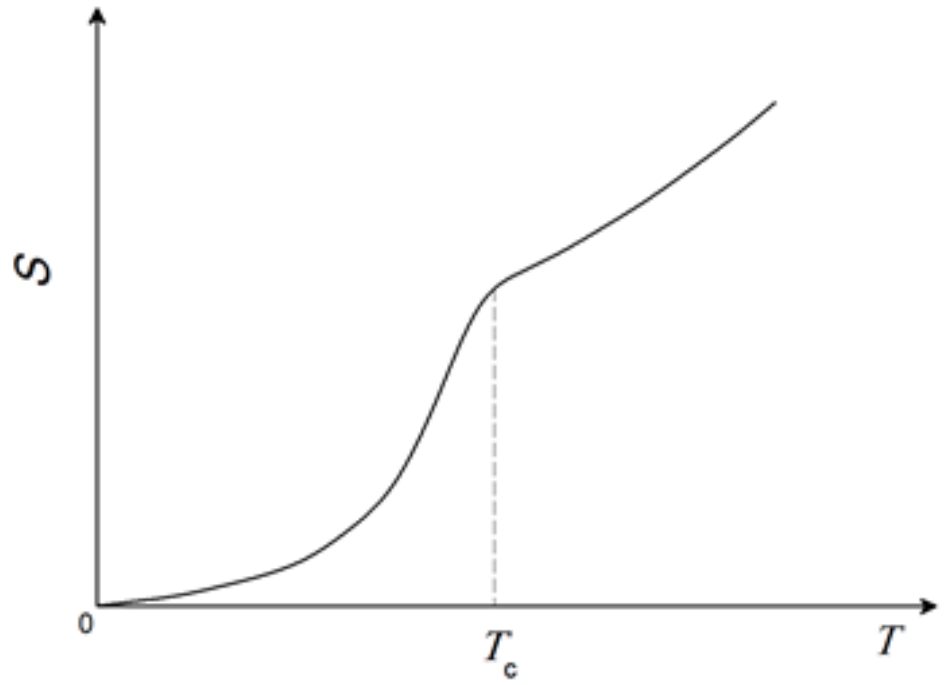


FIG. 1. Characteristic temperature dependence of the electron entropy $S_e(T)$ of superfluid Bose liquids of strongly coupled (polaronic) Cooper pairs in high-T_c cuprates.

III. CONCLUSION

An analytical expression has been derived for the temperature-dependent entropy, $S_{\rm e}(T)$, of a superfluid (superconducting) 3D Bose liquid in high-T_c cuprates. This study reveals that as the temperature decreases below the critical temperature T_c, the entropy $S_{\rm e}(T)$ experiences a sharp drop. This decline is attributed to the emergence of an energy gap, $\Delta_g(T)$, in the excitation spectrum of the Bose liquid, which signifies a transition into the superconducting phase. The formation of this energy gap restricts the number of thermally accessible states, leading to a significant reduction in entropy as the system enters the superconducting state.

Furthermore, the theoretical behavior of $S_{\rm e}(T)$ as predicted by this model aligns closely with experimental observations in high-T_c cuprates. Specifically, the temperature dependence of $S_{\rm e}(T)$ both above and below T_c exhibits characteristics that resemble the experimentally measured entropy behavior in high-T_c superconducting cuprates, such as $YBa_2Cu_3O_{6+x}$. This similarity provides additional validation for the model, suggesting that the proposed Bose-liquid approach accurately captures key aspects of the thermodynamic properties of high-T_c superconductors.

In conclusion, the obtained analytical expression for $S_{\rm e}(T)$ not only enhances our understanding of the entropy behavior in high-T_c cuprates but also supports the hypothesis of Bose-liquid superconductivity in these materials. The findings contribute to a broader understanding of the unique thermodynamic properties observed in high-temperature superconductors, potentially aiding the development of theoretical models that could explain the mechanisms underlying high-T_c superconductivity.

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