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The maximum flux of neutrons in thermal research reactors

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Abstract. The goal of this work is to determine the space distribution of fuel concentration in thermal reactors to maximize the flux of neutrons. This problem reduces to a nonclassical variational problem with some approximations, which can be solved with Pontryagin's maximum principle. It is demonstrated that the reactor core's ideal fuel configuration consists of an infinitely thick perimeter reflector. This refers to the region within the core where the rate of heat generation per unit volume (power density) is maintained at a safe and controlled level. Exceeding this limit could lead to overheating and damage to the reactor.

Keywords: neutron flux, Pontryagin maximum principle, power density, thermal neutron, moderator, reflector, variational problem.

I. INTRODUCTION

The current developments in research and reactor development are focused on boosting power, specifically power density, to meet the demand for high neutron flux. Ten years ago, The intensity of the thermal neutron flow rose from 3×10^{14} to 3×10^{15} n cm^{-2} sec^{-1}. A thermal neutron flux of up to 5×10^{16} n $cm^{-2}sec^{-1}$ has been built into reactors today. The high power density of reactors and the reflector-centered design of core have allowed for the achievement of very large thermal neutron fluxes. Different types of reactors operate at different power densities. For example, light-water reactors (LWRs) typically have mean power densities in the range of 30-100 kW/liter, while sodium-cooled fast reactors (SFRs) can reach much higher values, like 300-600 kW/liter.

For a randomly selected form of the fuel concentration space distribution, the neutron flow is proportional to the total reactor power and indirectly limited by several technological issues. Nevertheless, they can generally be decreased to the power density limit. These limitations may vary based on the reactor type and the construction design of its core. It is evident that, given the same temperature constraints, varying reactor core layouts equate to varied neutron flux values. For this reason, the topic of finding the fuel concentration distribution in space for which the reactor center's neutron flux is largest under identical thermal conditions is posed. Numerous works, including tangible implementations like the reactors discussed earlier with the reflector at the center, address the problem of figuring out the Fuel concentration distribution in space that, in a given set of circumstances, produces the maximum neutron flux. The shared flaw across the aforementioned strategies is that they rely on fuel concentration space distributions that are assumed and the optimal option is selected through comparison of the outcomes. One drawback of these direct approaches is that there are no well-defined optimality criteria, thus even the best solution cannot be deemed optimal. Here, we characterize an indirect method of solving the problem as follows: using the maximal neutron flux and taking into account thermal constraints, ascertain Fuel concentration distribution in space in the reactor core. According to mathematics, the issue as it has been formulated is variational. The reactor core's spatial distribution of fuel concentration, however, is outside the purview of the standard calculus of variations because it is a limited function that is piece - wise continuous. The current study demonstrates that Pontryagin's maximal principle,this problem can be resolved by applying the mathematical method of variations in a modern way [1].

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II. MODEL OF THE PROCESS

The well recognized hypothesis of two-group diffusion and the presumption regarding the feasibility of isolating the neutron flux's spatial distribution serves as the foundations for the model of the process of neutron appearance and absence in the reactor core used here([2]).

We categorize the fuel concentration effect on two-group parameters into weakly and strongly dependent functions based on its degree of intensity.The infinite multiplication factor is one of the fuel concentration's highly dependent functions k_∞(i.e., The cross section for atomic thermal neutron absorption normal utilization factor f) and the cross section for atomic thermal neutron absorption $\sum_{as}$.The other characteristics are only marginally affected by the fuel concentration, therefore they can be considered constant to a first order. Actually, in reactor systems using naturally occurring and slightly enriched uranium fuel, the resonance escape probability $\bar{p}$ depends significantly on the fuel concentration, particularly in thermal systems where the lattice distance is not small in comparison to the mean free route of neutrons. It has less of an impact on the form of the flux distribution in these zones and, consequently, the spread of fuel concentration form, but rather more of an impact on the sizes of the important system zones([3]).

Let us examine a reactor core with symmetric qualities that resembles a cylinder or parallelepiped. We examine a coordinate system where the origin is located at the center of core . Assume for the moment that the fuel concentration is constant in one direction and varies randomly in another, such as the direction of the z-axis.

The infinite multiplication factor and the very small absorption segment for the thermal spectrum of neutrons are provided by

$$\sum_{as}(z) = (1 + u(z)) \sum_{i} \sum_{ai} \tag{1}$$

and

$$k_\infty(z) = ku(z)(1 + u(z))^{-1} \tag{2}$$

where $k = \eta \in \bar{p}$, if someone explains the role $u(z) = \sum_{au}(z)(\sum_i \sum ai)^{-1}$, where $\sum_{au}$ and $\sum_i \sum_{ai}$ are the macroscopic thermal neutron absorbing spectra of uranium and other elements, respectively. $0 \leq u(z) \leq u_{max}$ is a class of limited and individually continuous functions, where the fuel concentration is proportional to $u(z)$.

Should someone present descriptions $z = Lt, \phi_s(t) = x_1(t), d\phi_s(t)/dt = x_2(t), \phi_f(t) = D_s D_f^{-1}(\bar{p})^{-1} x_3(t)$, and $d\phi_f(t)/dt = D_s D_f^{-1}(\bar{p})^{-1} x_4(t)$, the structure of the two-group transmission equations is as follows.:

$$\begin{aligned} dx_1(t)/dt &= x_2(t), \\ dx_2(t)/dt &= (u(t) + v^2)x_1(t) - qx_3(t), \\ dx_3(t)/dt &= x_4(t), \\ dx_4(t)/dt &= -ku(t)x_1(t) + \mu^2 x_3(t). \end{aligned} \tag{3}$$

where

$$v^2 = 1 + \alpha^2, q = L^2\tau^-1, \mu^2 = q + \alpha^2$$

Complete bending of the fuel intensity direction is represented by the parameter α^2, which is provided in *moderator L* diffusion length units, $[L^2 = D_s(\sum_i \sum_{ai})^{-1}]$, where τ is the moderator neutron age and all the core components are included, excluding the fuel. For the symmetric core, boundary conditions are

$$x_1(t_1) = x_2(0) = x_3(t_1) = x_4(0) = 0$$

where $2Lt_1$ is the reactor's height.

By appropriately substituting variables and adhering to the same assumptions, For a cylinder-shaped reactor with unchanging axial features or a spherical reactor, the two-group circulation model may also be simplified to Eqs. (3). When the orientations of continuous fuel concentration $(\alpha^2 = 0)$ have infinite dimension, Eq. (3) decreases to the form that is already covered in [4].

III. EXPRESSION OF THE PROBLEM MATHEMATICALLY

The highest thermal neutron flux $x_1(0)$ divided by the reactor power is used to determine a research reactor economy (refer to [5]).

$$P = K_1 \int_0^{t_1} u(t)x_1(t)dt$$

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The core shape determines the value of the constant K_1. Regarding a cylindrical reactor possessing consistent radial properties, its value is

$$K_{1c} = 4\pi j_{01} J_1(j_{01}) L^3 \alpha^{-2} \sum_{fu} \sum_{au}^{-1} \sum_{i} \sum_{ai}$$

since, according to Eq. (3),

$$x_1(0) = \int_0^{t_1} x_2(t)dt$$

the ratio $x_1(0)/P$ reaches its maximum for

$$I_1(t_1) = \int_0^{t_1} x_2(t)dt = Minimum$$

subject to

$$I_2(t_1) = \int_0^{t_1} u(t)x_1(t)dt = Const$$

One must consider the temperature constraints when developing a reactor.These restrictions are based on the material's resistivity, the hydraulic and thermal conditions, and other factors. The reactor's operational parameters are ultimately determined by these constraints. The reactor power density limit reduces the thermal constraints to a first approximation, that is, $dP/dV \leqslant (dP/dV)_{max}$.By introducing the parameter,

$$\gamma = max(u(t)x_1(t) = \sum_{au} (\sum_{fu} \cdot \sum_{i} \sum_{ai})^{-1} (dP/dV)_{max}$$

It is evident that the power density limit takes the following form, $u(t)x_1(t) - \gamma \leqslant 0$.

Consequently, the reduction of the problem of determining the maximum neutron flow in thermo reactors is represented by the variational problem that follows: finding the piecewise continuous and limited function $u(t)$ such that

$$I_1(t_1) = \int_0^{t_1} x_2(t)dt = Minimum$$

in the subsequent circumstances:

(a) differential equation system (3)

(b) $I_2(t_1) = \int_0^{t_1} u(t)x_1(t)dt = Const$

c) $u(t)x_1(t) - \gamma \leqslant 0$

(d) $0 \leqslant u(t) \leqslant u_{max}$

The isoperimetric condition $I_2(t_1)$ and the integral functional $I_1(t_1)$ might alternate, as the nature of this issue implies. The common variance calculus is not able to solve this variational problem. However, Pontryagin's maximal principle can be used to obtain a solution for u(t).

IV. SOLUTION OF THE VARIATIONAL PROBLEM

We will utilize the findings of the contemporary calculus of variations, specifically to find the optimal function $u(t)$. Equations (3) and the subsequent equations make up the entire set of Equations of difference for stage coordinates:

$$\begin{aligned} dx_0(t) &= x_2(t) \\ dx_5(t)/dt &= u(t)x_1(t) \\ dx_6(t)/dt &= 1 \end{aligned} \tag{4}$$

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The limits that these phase coordinates are $x_0(0) = x_5(0) = x_6(0) = 0$ and $x_5(t_1) = const > 0$.

The model for process (3) can be found in a system of differential equations with self-sufficient form. Additionally, a phase coordinate $x_6(t) = t$ has been introduced. This allows for the explicit expression of the implicit limitation $p(x_i(t), u(t)\, x_1(t) - \gamma \leq 0$ in the form of $g(x_1(t)) = x_5(t) - \gamma x_6(t) + Const$. Later on, while utilizing the jump conditions for the auxiliary functions, a limitation of the type $g(x_i(t))$is needed [6][7].

The way for the Hamiltonian is expressed by

$$H(t) = \sum_{i=0}^{6} \psi_i(t)(dx_i(t)/dt) = h(t) + u(t)x_i(t)v(t) \tag{5}$$

where

$$h(t) = (\psi_0 + \psi_1)x_2 + \psi_2(\nu^2 x_1 - qx_3) + \psi_3 x_4 + \mu^2 \psi_4 x_3 + \psi_6 \tag{6}$$

$$v(t) = \psi_2 - k\psi_4 + \psi_5 \tag{7}$$

Here, the functions $\psi_i(t)$ are the so-called auxiliary functions, given by the following system of differential equations:

$$\begin{aligned} d\psi_0(t)/dt &= 0 \\ d\psi_(t)/dt &= -\nu^2\psi_2(t) + u(t)(\lambda(t) - v(t)) \\ d\psi_2(t)/dt &= -\psi_0(t) - \psi_1(t) \\ d\psi_3(t)/dt &= q\psi_2(t) - \mu^2\psi_4(t) \\ d\psi_4(t)/dt &= -\psi_3(t) \\ d\psi_5(t)/dt &= 0 \\ d\psi_6(t)/dt &= 0 \end{aligned} \tag{8}$$

where

$$\lambda(t) = \begin{cases} v(t) & \text{for } u(t)x_1(t) \equiv \gamma \\ 0 & \text{for } u(t)x_1(t) < \gamma \end{cases}$$

The following boundary conditions result from the transversally criterion (see Theorem 3 of Ref. 1): $\psi_1(0) = \psi_2(t_1) = \psi_3(0) = \psi_4(t_1) = \psi_6(t_1) = 0$

The Hamiltonian and the series of formulas for the phase measurements and auxiliary function may be obtained using the same mathematical approach if the integral functional $I_1(t_1)$ and the isoperimetric condition $I_2(t_1)$ switch places. The interchange between $\psi_0(t)$ and $\psi_5 t)$ is the only distinction. Therefore, it follows that $\psi_5(t) = C\psi_0(t), C > 0$, i.e., $\psi_5(t) < 0$.par It is evident that $u(t) = \gamma x_1^{-1}(t)$ must be the case for those sub-intervals in which the criteria for $u(t) = x_1(t) \equiv \gamma$ is completely satisfied. In other words, a reactor zone that has a constant allowable power density should be one sort of radiation zone. After examining the Hamiltonian (5) for $u(t)x_1(t) < \gamma$, we deduce that $u(t) = u_{max}$ is required in sub-intervals where $v(t) > 0$; nevertheless, for $v(t) < 0$, it must be as $u(t) = u_{min} = 0$. Consequently, a reflector zone with zero fuel concentration and a zone with the highest energy concentration are obtained as additional reactor zone types.

In addition to these kinds of reactor zones, there is another scenario where the classical calculus of variations-derived requirement $\partial H/\partial u = 0$ holds true. This situation does not apply to this problem, though, as the auxiliary functions admit an easy solution for $v(t) \equiv 0$.

All possible zone configurations for the ideal core have been considered in the way described above. The number and order of the reactor zones are not precisely stated in the maximal concept, so the optimal $control$ $u(t)$ has not been fully established. We will employ the constraints on The energy concentration $u(t)x_1(t)$ and the function $u(t)$, as well as the boundary conditions for x_i and ψ_1.

We obtain $u(0) = 0$ by applying the boundary conditions at $t = 0$ along with the symmetry conditions. This indicates that The first sector of the ideal core design is the reflector form (center reflector). We discover that $v(t) < 0$ in the vicinity of $t = t_1$, using the boundary criteria as before. This indicates that The last zone has a peripheral reflector, which is a similar sort of reflector. Additionally, it has been demonstrated that the peripheral reflector's thickness is unlimited [8][9][10]. In conclusion, we find that the sector of type $u(t) = \gamma x_1^{-1}$ comes after the main reflector, and the sector of type $u(t) = u_{max}$ comes before the peripheral reflector; the function $u(t)$ has to be continuous at the boundary of these sectors. Based on the limitation to $u(t)$ and $u(t)x_1(t)$, this is stated. Consequently, the following (Fig. 1) is the most likely contender for the ideal fuel concentration distribution, or the function $u(t)$:

$$u(t)/u_{max}(t) = \begin{cases} 0, & 0 \leq t < h_1 \\ x_1(h_2)/x_1(t), & h_1 \leq t \leq h_2 \\ 1, & h_2 \leq t \leq h_3 \\ 0, & h_3 < t \leq \infty \end{cases} \tag{9}$$

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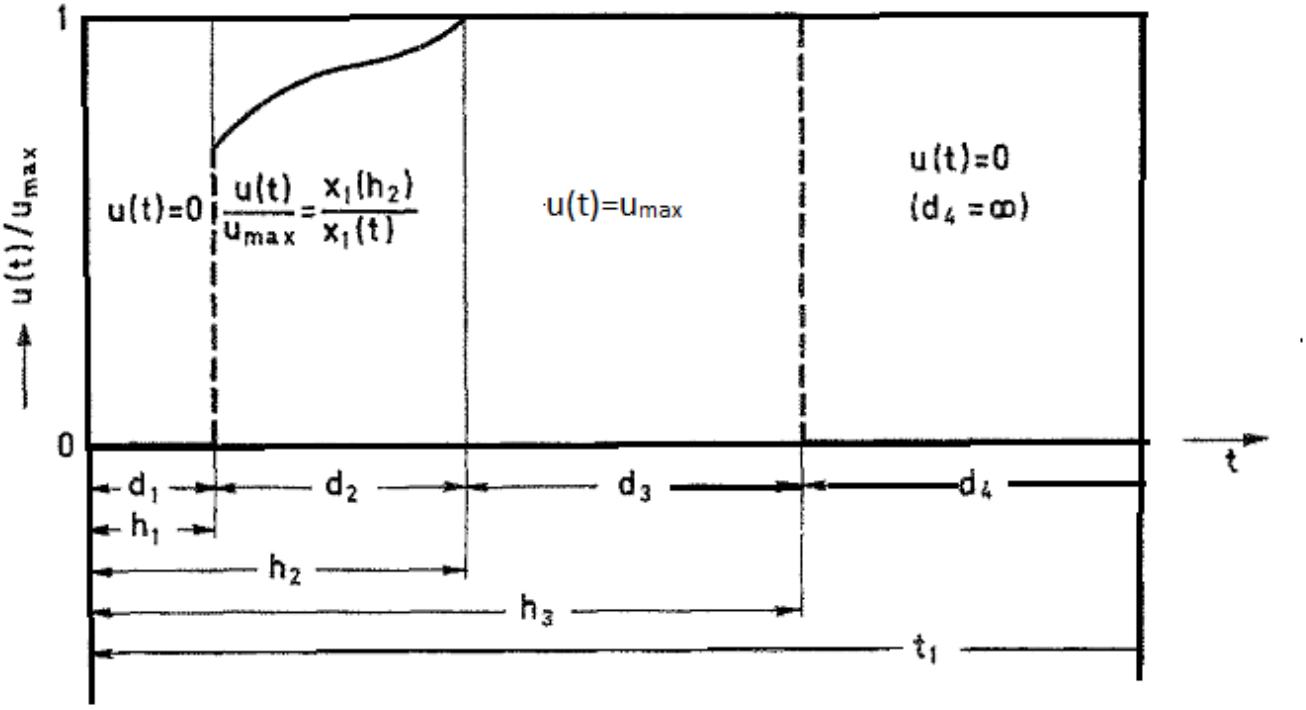


FIG. 1. Optimal core configuration

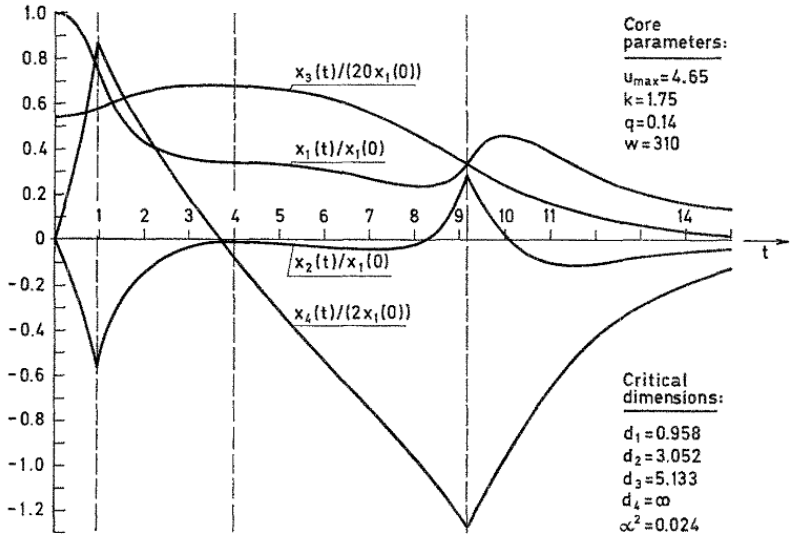


FIG. 2. Solution for $x_i(t)$

V. VERIFICATION OF THE IDEAL PARAMETERS

It is important to first calculate the functions $x_i(t)$ and $\psi_i(t)$, as well as the crucial thicknesses d_1, d_2, d_3 (Fig.1), in order to verify the optimality criteria. Due to their extent, solutions for $x_i(t)$, $\psi_i(t)$, and the critical thicknesses are not provided here. We merely demonstrate their acquisition.

Finding answers for $x_i(t)$ is simple when using the functions $u(t)$ provided in (9).

Every reactor zone has a different set of solutions. The combination of constants is computed using the given continuity condition and threshold requirements for these functions. There are still two fundamental equations where the thicknesses

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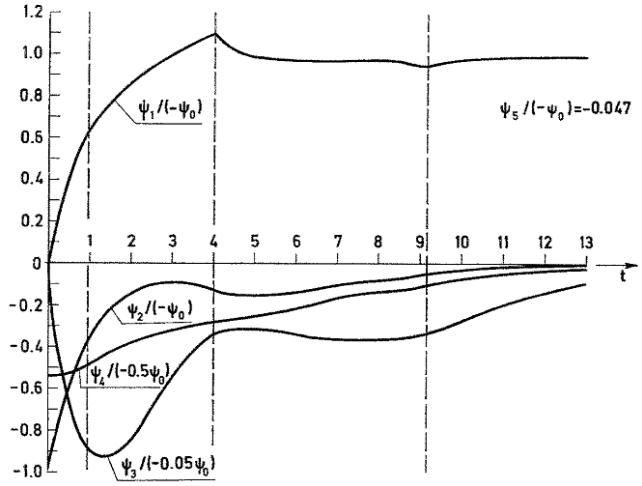


FIG. 3. Solution for $\psi_i(t)$

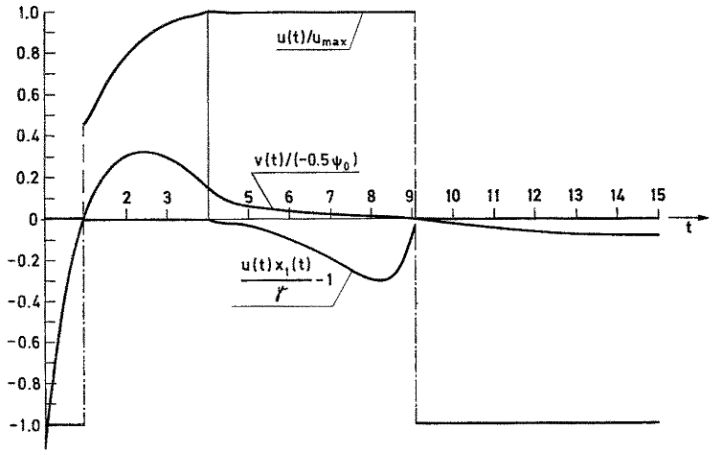


FIG. 4. Solution for $u(t), v(t)$and $u(t)x_1(t)$

d_1, d_2, d_3 are the only unidentified factors. This is due to the fact that more equations—two more than the quantity of integrative permitted power density parameters and supplied power. Instead of using the supplied power and allowable power density, it would be more convenient to utilize the condition of given power P and its proportion to the given permissible power density $(dP/dV)_max$ in the form $\omega = K_2 L^{-3}(dP/dV)^{-1}_{max}$. Here, K_2 is a constant that adds up to $K_{2c} = (4\pi J_1(j_{01}))^{-1} = 0,06374$ for the cylindrical reactor with uniform radial features. Considering this, we lessen the isoperimetric requirement to

$$I_2(t_1) = \int_0^{t_1} u(t)x_1(t)dt = \omega\gamma\alpha^2.$$

Similarly, the auxiliary functions $\psi_i(t)$ are determined. With the possible exception of $t = h_1$, they are continuous at every point along the independent variable t. On the other hand, one finds that $\lambda(h_1) = 0$, i.e., $V(h_1) = 0$, and that every additional role must likewise be continuous at this point if the previously indicated constraint $g(x_i)$ and the transition condition (Ref. 1) are implemented. $\psi_6(t) \equiv 0$ and $v(h_3) = 0$ are the outcomes of all the auxiliary functions with complete continuity. The homogeneous nature of the differential equations (8) allows for the normalization of the system using any positive constant, such as - ψ_0. By applying the formulas, $v(h_1) = V(h_3) = 0$. Limitation and continuity conditions are used to find the connection constants in the solutions for $\psi_i(t)$. There is still one transcendental equation left, in which the only unknowns are the thicknesses (d_1, d_2, d_3); this equation, together with the two previously found equations, constitutes a collection of three fundamental equations. These sets of equations may be solved repeatedly to provide the critical sizes d_1, d_2, d_3. The phase parameters and additional tasks are solved for the situation shown in Figures 2-3. and Figure 4 shows the associated function

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$u(t)$. The Hamiltonian (5) is consistent and continuous at all times t according to this solution form. As $H(\infty = 0$, $H(t) = O$ must be the case.

Additionally, zero must be demonstrated to be the supreme of the Hamiltonian at the same time (5). It suffices to verify that the sign of function $v(t)$ in Fig. 4 matches the character of each zone $[v(t) > 0$ for $h_1 < t < H_3]$ and $0 \leq t < h_1$; and $v(t) < 0$ for $0 \leq t < h_1$ [11][12].

It is demonstrated that Theorem 22 of Reference 1 also satisfies requirements (c) and (d). It is important to note that the preceding theorem's condition (c) states that $\lambda(t) \geq 0$ [13]], but not $d\lambda(t)/dt \leq 0$. It is clear from Fig. 4 that $u(t)$ and $u(t)x_i(t)$ fall within acceptable bounds. Since all of the requirements of Pontryagin's maximum principle are met in this instance, the fuel concentration distribution form provided by Eq. (9) is indeed optimal [14][15].

VI. CONCLUSIONS

It is shown that determining the greatest neutron flow in thermal nuclear reactors is fundamentally a matter of solving a variational problem in which the spatial distribution of the fuel proportion is unknown. This issue is difficult to answer with the conventional calculus of variations due to its character. This is due to the reactor's restricted fuel concentration and power density.

The issue of the variable has been solved through the application of Pontryagin's maximal principle. An explicit analytical form of the fuel concentration optimum distribution has been obtained. It has to be mentioned that the outcomes are reliable on the assumption that each zone of the core's ideal configuration has the same moderation qualities.

The neutron's two-group diffusion equation serves as the foundation for the process approach and is oriented in the plane geometry (axial direction) of a cylindrical reactor. Nonetheless, a comparable model may be created for reactors with spherical and cylindrical geometries by appropriately substituting variables; in this case, the best core arrangement (Fig. 1) can also be applied to these reactors.

The answer for the ideal fuel concentration can not be obtained analytically since the precise handling of resonance absorption, which is roughly considered here, necessitates the use of numerical methods.

The method described here for determining the conditions under which the neutron flux achieves a maximum can also be used in the situation of varying moderation qualities of each reactor zone. Nevertheless, there is no analytical solution for the ideal fuel concentration. It might be advantageous in some circumstances to use materials with distinct moderating qualities for the core and periphery reflectors.

Let us conclude by pointing out that, of all the known solutions to the problem under consideration, the one implemented in the light-water, high-flux reactor HFIR is the closest to ideal.

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