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Branching intra-/inter-band reduced E2-transition probabilities in the model free triaxiality

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In the framework free triaxility model branching ratio reduced E2-transitions probabilities are studied for the full region changes of the triaxility parameter. The variables of quadrupole deformations β_2 and γ are dynamic. The Davidson potential for β_2 and γ variables was being used. The wave function of the free triaxility model was being obtained by taking into account the high-order terms of the rotational energy operator series. The quadrupole moment of the nucleus expressed in terms of two internal quadrupole moments: Q_2 is quadrupole moment about the symmetry axis, Q_2 is the measure of the shape asymmetry about the symmetry axis. The intra/inter-band reduced E2-transition probabilities ground-state-band, β- and γ-bands are being presented and are being expressed in terms of four parameters: μ_β, μ_γ, γ_0 and Q_0. The influence of γ-oscillations to the reduced probabilities of E2-transitions takes into account are numerically. The sensitivity of the intra-/inter-band reduced E2-transition probabilities to the triaxiality parameter was analyzed.

Keywords: free triaxiality, quadrupole deformations, ground-state-, β- and γ-bands, intra/inter-band $E2$-transition probabilities.

I. INTRODUCTION

Quadrupole shapes are described by the parameters β_2 and γ for the axial deformation and the deviation from axiality [2–8]. In Ref.[2] was proposed the free triaxialty model, where deformation parameters β_2 and γ taking into account are dynamically. In Ref.[4], a comparative analysis of some non-adiabatic models [2, 3, 9–13] was carried out, and it was determined that the free triaxiality model reproduces the experiment better than other models. In Ref.[5], the free triaxiality model the high-order terms of the rotational energy operator series were being considered. It is in Ref.[14] leads to improve considerably the agreement of the results with experimental data.

Multipole electrical transitions are associated with the multipole electric moment of the nucleus, which is associated with the distribution of the proton density in the nucleus. For quadrupole, octupole, and higher multipole transitions, in the sharp step approximation for density distribution, the multipole moment is related to surface deformation parameters. In Ref.[15] intra-/inter-band reduced E2-transitions probabilities in excited collective states of even-even lanthanide and actinide nuclei was studied. In Ref.[16] one-parameter Davydov-Filippov model have been used often to study the inter γ-ground-bands B(E2) ratios for the triaxially deformed nuclei. But in this work variables β_2 and γ are statistic. And consideration of the intra-/inter-band reduced E2-transitions probabilities are restricted by low-spin states. In this work we will consider variables β_2 and γ are dynamic. The intra-/inter-band reduced E2-transitions probabilities we consider by including high spin states [11, 12]. Comparison of these data with the properties of the energy spectrum can provide information about the nature of high moment states for nuclei in various regions of the periodic table.

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II. REDUCED PROBABILITIES OF *E*2 TRANSITIONS BETWEEN EXCITED COLLECTIVE STATES OF NONAXIAL EVEN-EVEN NUCLEI

The reduced probabilities of $E2$-transitions between the $n'_\gamma n'_{\beta_2} I'\tau'$ and $n_\gamma n_{\beta_2} I\tau$ [4] states are given by

$$B[E2; n_\gamma n_{\beta_2} I\tau \to n'_\gamma n'_{\beta_2} I'\tau'] = \frac{5}{16\pi(2I+1)} \sum_{MM'\mu} | < n'_\gamma n'_{\beta_2} I'\tau' | Q_{2\mu} | n_\gamma n_{\beta_2} I\tau > |^2, \tag{1}$$

where

$$Q_{2\mu} = \frac{\beta_2}{\beta_0} \left[Q_0 D^2_{\mu 0} \cos\gamma + Q_2 \left(D^2_{\mu 2} + D^2_{\mu,-2} \right) \frac{\sin\gamma}{\sqrt{2}} \right], \tag{2}$$

where Q_0 is the quadrupole moment about the symmetry axis, Q_2 is the measure of the shape asymmetry about the symmetry axis [17]. For a system with an ellipsoidal shape, the quadrupole moment of the nucleus can be expressed in terms of two internal quadrupole moments Q_0 and Q_2. By the by [17] the ratio of Q_2 and Q_0 is a measure of the deviation from symmetry about the 3 axis, which is depends in terms of the parameter γ_0

$$Q_2 = \frac{\tan\gamma_0}{\sqrt{2}} Q_0$$

Reduced probabilities of $E2$-transitions between $i \equiv \{n_\gamma n_{\beta_2} I\tau\}$ and $f \equiv \{n'_\gamma n'_{\beta_2} I'\tau'\}$ can be represented as

$$B[E2; i \to f) = B_a[E2; I\tau \to I'\tau'] S^2_{if\beta_2} S^2_{if\gamma}, \tag{3}$$

where $B_a[E2; I\tau \to I'\tau']$ – probabilities of $E2$-transitions in excited states of a symmetric rotator [2] and

$$S_{if\beta_2} = \int_0^\infty F_i(\beta_2) \frac{\beta_2}{\beta_0} F_f(\beta_2) \beta_2^4 d\beta_2, \tag{4}$$

which takes into account the deformability of an even-even nucleus. The matrix element $S_{if\gamma}$ has the form

$$S_{if\gamma} = \int_o^\infty \xi_i(\gamma)\phi_i(\theta_i) \left\{ Q_0 D^2_{\mu 0} \cos\gamma + Q_2 \left(D^2_{\mu 2} + D^2_{\mu,-2} \right) \frac{\sin\gamma}{\sqrt{2}} \right\} \times$$

$$\times \xi_f(\gamma)\phi_f(\theta_i) |\sin 3\gamma| \sin\theta_2 d\gamma d\theta_1 d\theta_2 d\theta_3. \tag{5}$$

takes into account the influence of γ-oscillations on the reduced probabilities of $E2$-transitions can be rewritten as follows:

$$S_{if\gamma} = \left\{ Q_0 \sqrt{2I+1} (I2K0|I'K') J_1 + Q_2 \sqrt{\frac{2I+1}{2}} \left[\sqrt{\frac{1+\delta_{K0}}{1+\delta_{K'0}}} (I2K2|I'K') + \right. \right.$$

$$\left. \left. + \sqrt{\frac{1+\delta_{K'0}}{1+\delta_{K0}}} (I2K-2|I'K') \right] J_2 \right\} \frac{2}{\lambda[\pi^2/(9b^2_{\gamma_0}), p+3/2] b^{p+p'+3}_{\gamma_0}}, \tag{6}$$

where $\lambda(x,a)$ is an incomplete Gamma function; $(I2K0|I'K')$–Clebsch-Gordon coefficients,,

$$J_1 = \int_0^{\pi/3} \gamma^{p+p'+2} \exp\left(-\frac{\gamma^2}{b^2_{\gamma_0}} \right) \cos\gamma d\gamma, \tag{7}$$

$$J_2 = \int_0^{\pi/3} \gamma^{p+p'+2} \exp\left(-\frac{\gamma^2}{b^2_{\gamma_0}} \right) \sin\gamma d\gamma. \tag{8}$$

The integrals J_1 and J_2 are calculated numerically.

The general expression for matrix elements $S_{if\beta_2}$ is very cumbersome. Therefore, we consider special cases.

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The reduced $E2$-transitions probabilities in the ground-state-band, where n_{β_2}=0, n'_{β_2}=0, n_γ=0 and n'_γ=0:

$$S_{if\beta_2} = \mu\sqrt{\frac{1}{\Gamma(q+\frac{7}{2})\Gamma(q'+\frac{7}{2})}}\Gamma\left(\frac{q+q'+7}{2}\right). \tag{9}$$

where $\Gamma(x)$–Gamma function.

The reduced E2-transitions probabilities between the ground-state- and β-band, where n_{β_2}=1, n'_{β_2}=0, n_γ= 0 and n'_γ=0:

$$S_{if\beta_2} = \frac{\mu}{4}\sqrt{\frac{1}{(q+\frac{15}{2})(q'+\frac{15}{2})\Gamma(q+\frac{7}{2})\Gamma(q'+\frac{7}{2})}}\Gamma\left(\frac{q+q'+8}{2}\right)\times$$

$$\times[(q+q'+10)(q+q'+8)-2(q+q'+3)(q+q'+8)+(2q+3)(2q'+3)]. \tag{10}$$

The reduced $E2$-transitions probabilities in the β -band, where n_{β_2}=1, n'_{β_2}=1, n_γ= 0 and n'_γ=0 :

$$S_{if\beta_2} = \frac{\mu}{2}\sqrt{\frac{1}{(q+\frac{15}{2})\Gamma(q+\frac{7}{2})\Gamma(q'+\frac{7}{2})}}\Gamma\left(\frac{q+q'+8}{2}\right)(q-q'-5). \tag{11}$$

The intra/inter-band reduced $E2$-transition probabilities are expressed in terms of four parameters: μ_β, μ_γ, γ_0 and Q_0. Note that the parameters μ_β, μ_γ and γ_0 are obtained from the description of the energy levels of excited states. The parameter Q_0 were obtained from a comparison of the calculated intra-/inter-band reduced $E2$-transitions probabilities with experimental data.

It is convenient to consider the ratio of the reduced probabilties of $E2$-transitions to the reduced probability of an $E2$-transition from the state of the first excited level with spin 2^+ to the ground state, which is given by:

$$b[E2; n'_\gamma n'_\beta I_i\tau' \to n_\gamma n_\beta I_f\tau] = \frac{B[E2; n'_\gamma n'_\beta I_i\tau' \to n_\gamma n_\beta I_f\tau]}{B[E2; 0021 \to 0001]}. \tag{12}$$

The basic shape properties of atomic nuclei are described through parameter of quadrupole deformation β_2 and axial-asymmetry parameter γ_0 [17]. The parameter β_2 is related to the intrinsic quadrupole moment and describes the magnitude of departure from the spherical shape along the principal axis in the intrinsic frame. At γ_0 = 0 and γ_0=π/3 it corresponds to the axially symmetric prolate and oblate shapes, respectively. The intermediate values of γ_0, within 0< γ_0 < π/3, give the triaxial shapes [8, 17, 18].

Here we will study various permitted E2-transitions, depending on the triaxiality parameter γ_0. At the same time, we take into account the entire range of changes in the parameter γ_0. All these ratios of the reduced E2-transitions probabilities (12) presented in figs.1– 12. The behavior of these ratios of the reduced E2-transition probabilities in more large values of the parameter γ_0 to explain is difficult. The major reason of this behavior is an anomalous behavior moment of inertia of the nucleus at large values of the parameter γ_0. But this effect is a topic for a separate discussion, so we skip it.

A. Intra/inter-band reduced E2-transition probabilities

In fig. 1-5 intra-band reduced $E2$-transition probabilities ground-state-band, β- and γ-bands are presented. At the same time, in fig. 3-5 the $E2$-transitions between even, odd and mixed spins for γ-band separately are represented.

In fig. 1 intra-band reduced $E2$-transition probabilities in ground-state-band are presented. These transitions at values $0^0 \leq \gamma_0 \leq 20^0$ of the parameter γ_0 are insensitive to changes in this parameter, while at values $20^0 \leq \gamma_0 \leq 30^0$ of the parameter the sensitivity of the transitions increases. These transitions are observed in the collective excitation spectra of almost all heavy deformed even-even nuclei, including high spin states. From fig. 1 obviously, that the influence deformation parameter μ_β to $E2$-transitions is not sensitive. Because this band is mostly rotational.

Behavior intra-band reduced $E2$-transition probabilities for ground-state-band, β-band are similar, but values of the $E2$-transition probabilities in this band are much larger than in the ground-state-band. These transitions at small values of the γ_0 parameter are also insensitive to changes in this parameter. And their amplitude values are slightly different on the variable parameter γ_0. In fig. 2 obviously, that the influence β_2-deformation to $E2$-transitions is more sensitive. The values of these $E2$-transitions decreases with increasing deformation parameter μ_β. Their amplitude values decreases with increasing deformation parameter μ_β. b(E2,$2I_\beta \to (2I-2)_\beta$) transitions are observed in the collective excitation spectra of most heavy deformed even-even nuclei. Because this band is mostly vibrational-rotational.

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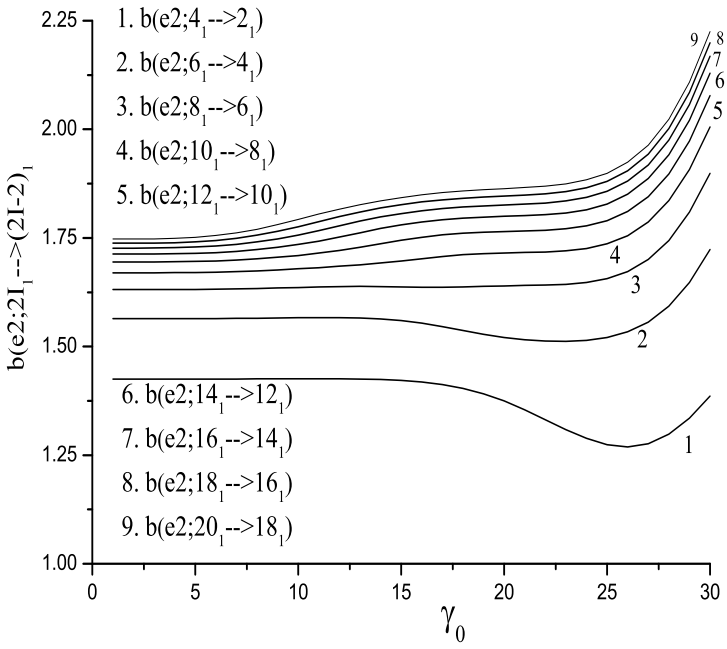


FIG. 1. Dependency reduced b(E2,$2I_1 \rightarrow (2I-2)_1$)-transition probabilities on the parameter γ at Q_0=9, Q_2=2, μ_γ=2, μ_β= 0.3. Here and in all figures I=1,2,3...

Intra-band reduced $E2$-transition probabilities in γ-band between even spins have more complex behavior. These transitions at small values of the γ_0 parameter are mostly sensitive to changes in this parameter. In fig. 3 obviously, that the influence β_2-deformation to $E2$-transitions is more sensitive. The values of these $E2$-transitions increases with increasing deformation parameter μ_β. These transitions are observed in the collective excitation spectra of most heavy deformed even-even nuclei: ^{164,166,168}Er [19]. Because this band is vibrational-rotational.

Intra-band reduced $E2$-transition probabilities in γ-band between odd spins have more complex behavior. In fig. 4 obviously, that the influence β_2-deformation to $E2$-transitions is more sensitive. The values of these $E2$-transitions increases with increasing deformation parameter μ_β. These transitions are observed in the collective excitation spectra of most heavy deformed even-even nuclei: ^{166,168}Er [19]. Because this band is vibrational-rotational. Note, from fig. 3 and 4 obviously, that the behavior reduced $E2$-transitions probabilities intra odd and even spins of the γ band are different.

On fig. 5 the behavior of the intra-band reduced $E2$-transitions probabilities between $(2I+1)_\gamma \rightarrow 2I_\gamma$ is presented. B(E2,$(2I+1)_\gamma \rightarrow 2I_\gamma$) are more sensitive to changes of the parameter γ_0. These transitions are observed in the collective excitation spectra of most heavy deformed even-even nuclei: ^{152}Sm and ^{166,168}Er [19]. In fig. 6-12 inter-band reduced $E2$-transition probabilities between ground-state-band and β-band, ground-state-band and γ-band, β-band and γ-band are presented. In fig. 6 behavior of the inter-band reduced $E2$-transitions probabilities b(E2,$2I_\gamma \rightarrow (2I-2)_1$) is presented. b(E2,$2I_\gamma \rightarrow (2I-2)_1$) are more sensitivity at values $20^0 \leq \gamma_0 \leq 30^0$ of the parameter γ_0. These transitions are observed in the collective excitation spectra of most heavy deformed even-even nuclei: ^{150}Nd, ^{152,154}Sm, ^{156,158}Dy, ^{156}Gd, ^{162,164,166,168}Er, ^{188}Yb, ^{230,232}Th, ^{234,238}U [19]. In fig. 7 behavior of the inter-band reduced $E2$-transitions probabilities between same spins $2I_\gamma \rightarrow 2I_1$ is presented. These transitions are observed in the collective excitation spectra of the following deformed even-even nuclei: ^{150}Nd, ^{156,158}Dy, ^{156}Gd, ^{162,164,166,168}Er, ^{188}Yb, ^{230,232}Th, ^{234,238}U [19]. In fig. 8 behavior of the inter-band reduced $E2$-transitions probabilities between $2I_\beta \rightarrow (2I-2)_1$ is presented. These transitions are observed in the collective excitation spectra of the following deformed even-even nuclei: ^{150}Nd, ^{152,154}Sm, ^{156,158}Dy, ^{156}Gd, ^{162,164,166,168}Er, ^{188}Yb, ^{230,232}Th, ^{234,238}U [19]. In fig. 9 behavior of the inter-band reduced $E2$-transitions probabilities between same spins $2I_\beta \rightarrow 2I_\gamma$ is presented. These transitions are observed in the collective excitation spectra of the following deformed even-even nuclei: ^{152}Sm, ^{168}Er [19]. In fig. 10 behavior of the inter-band reduced $E2$-transitions probabilities between same spins $2I_\gamma \rightarrow (2I-2)_\beta$ is presented. These transitions are observed in the collective excitation spectra of the following deformed even-even nuclei: ^{152}Sm,^{156}Gd [19]. In fig. 11 behavior of the inter-band reduced $E2$-transitions probabilities between same spins $(2I+1)_\gamma \rightarrow 2I_1$ is presented. These transitions are observed in the collective excitation spectra of the following deformed even-even nuclei: ^{152}Sm, ^{156}Gd, ^{166,168}Er [19]. In fig.

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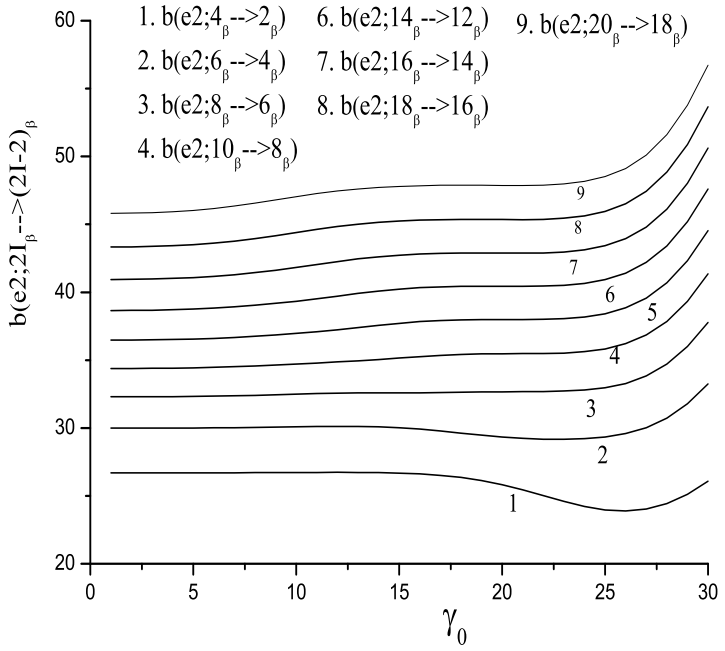


FIG. 2. The same as in Fig. 1, but for b(E2,$2I_\beta \rightarrow (2I-2)_\beta$).

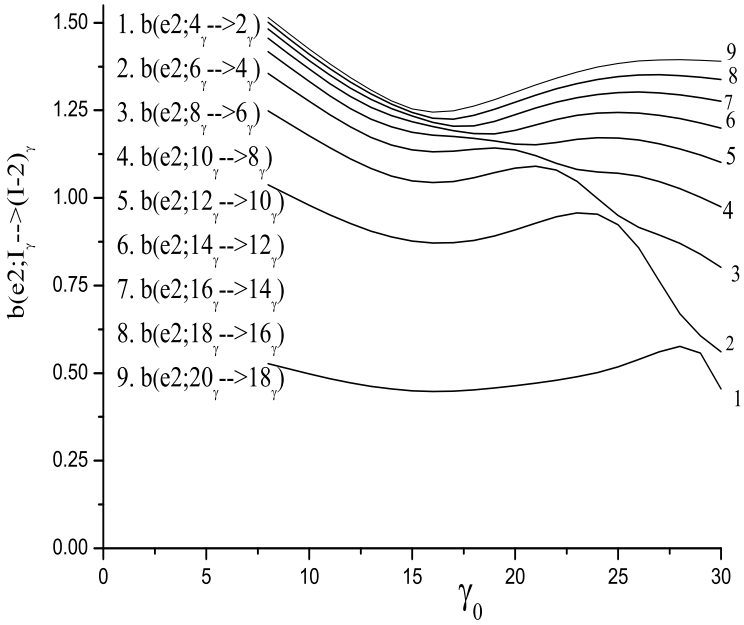


FIG. 3. The same as in Fig. 1, but for b(E2,$2I_\gamma \rightarrow (2I-2)_\gamma$) between even spins.

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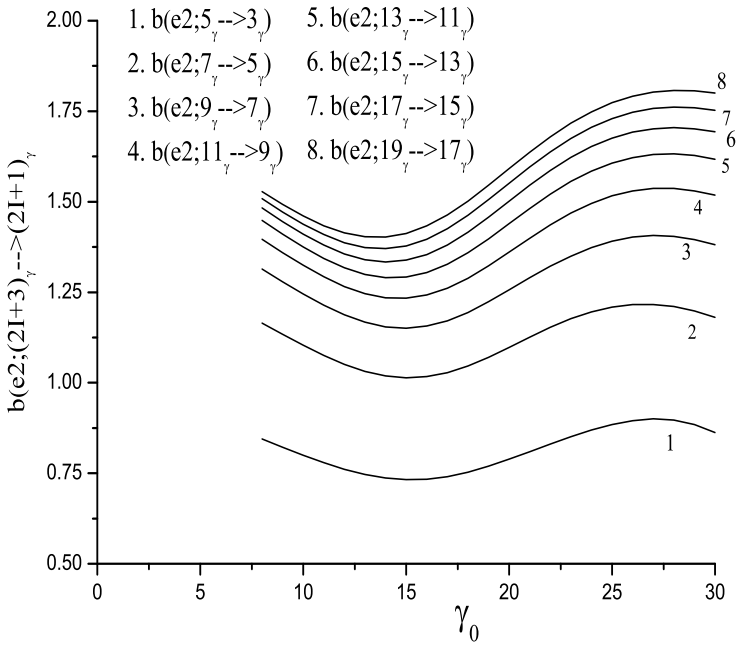


FIG. 4. The same as in Fig. 1, but for b(E2,$(2 \cdot I + 3)_\gamma \rightarrow (2 \cdot I + 1)_\gamma$) between odd spins.

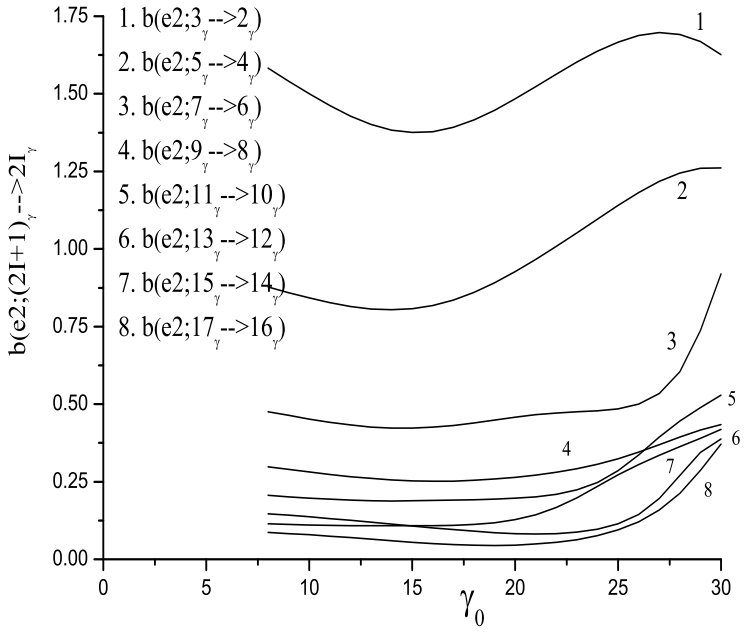


FIG. 5. The same as in Fig. 1, but for b(E2,$(2I + 1)_\gamma \rightarrow 2I_\gamma$).

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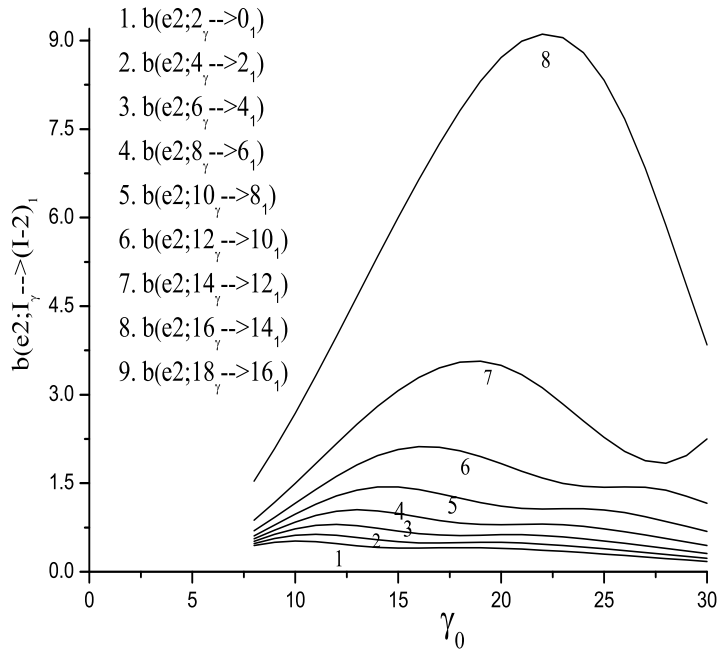


FIG. 6. The same as in Fig. 1, but for b(E2,$2I_\gamma \rightarrow (2I-2)_1$).

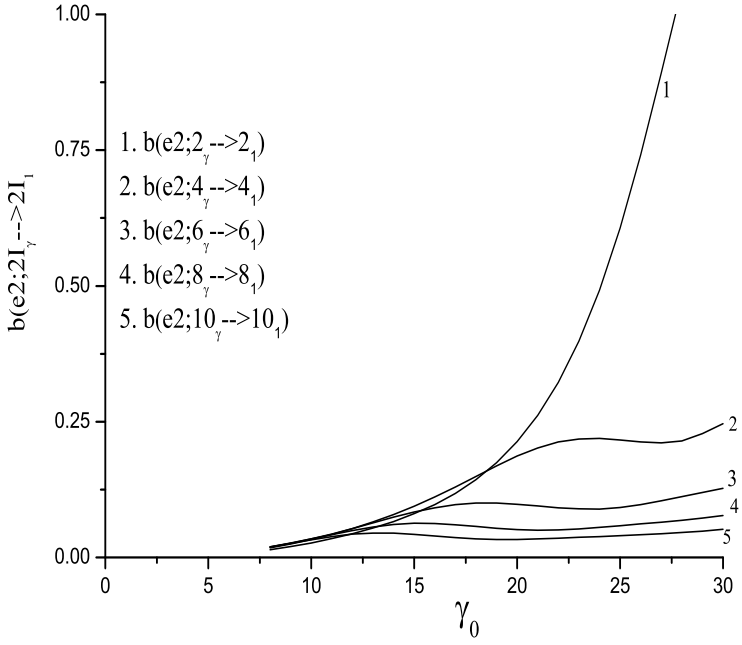


FIG. 7. The same as in Fig. 1, but for b(E2,$2I_\gamma \rightarrow 2I_1$).

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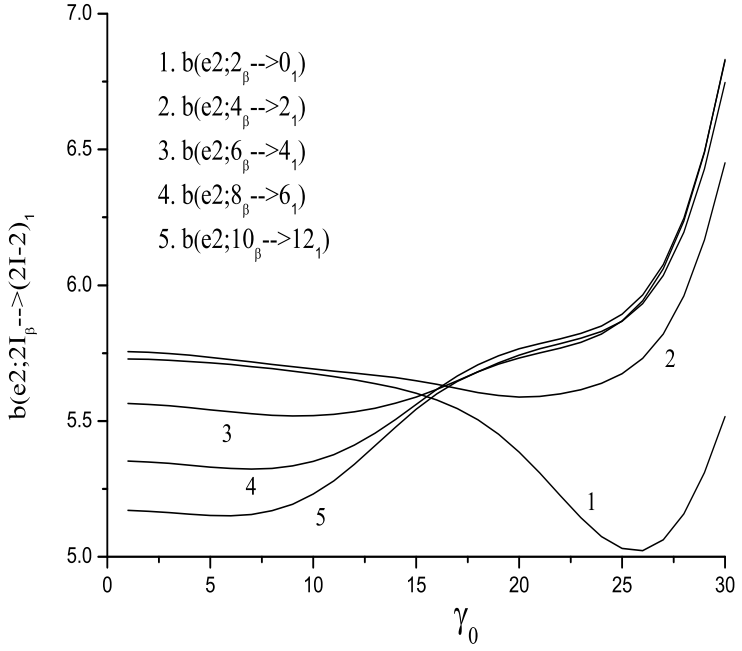


FIG. 8. The same as in Fig. 1, but for b(E2,$2I_\beta \rightarrow (2I-2)_1$).

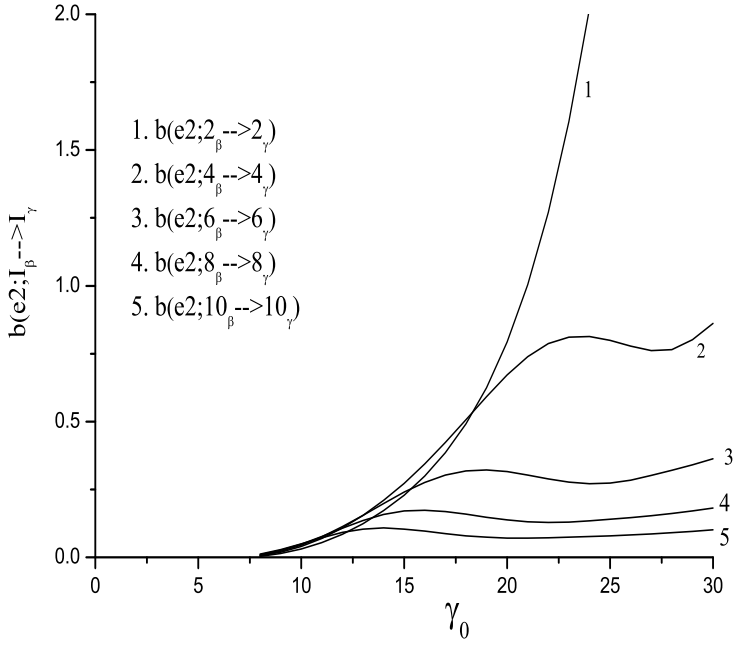


FIG. 9. The same as in Fig. 1, but for b(E2,$2I_\beta \rightarrow 2I_\gamma$).

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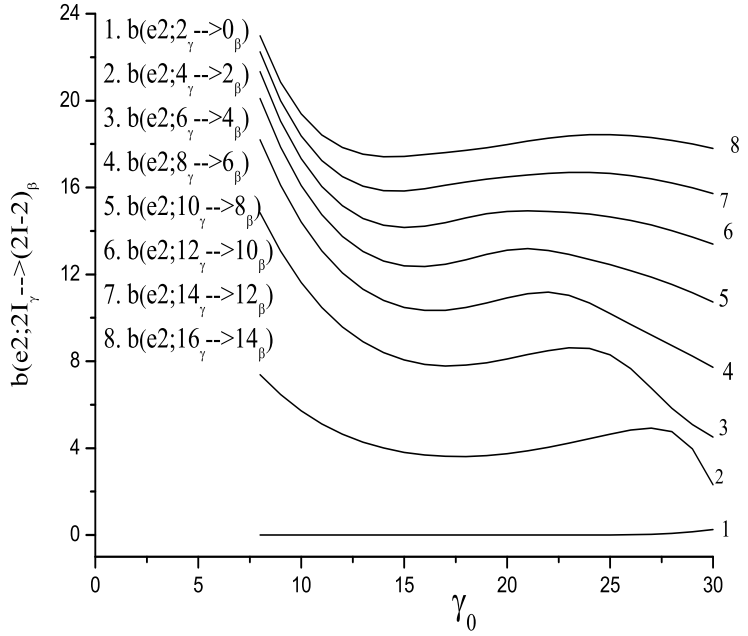


FIG. 10. The same as in Fig. 1, but for b(E2,$2I_\gamma \rightarrow (2I-2)_\beta$).

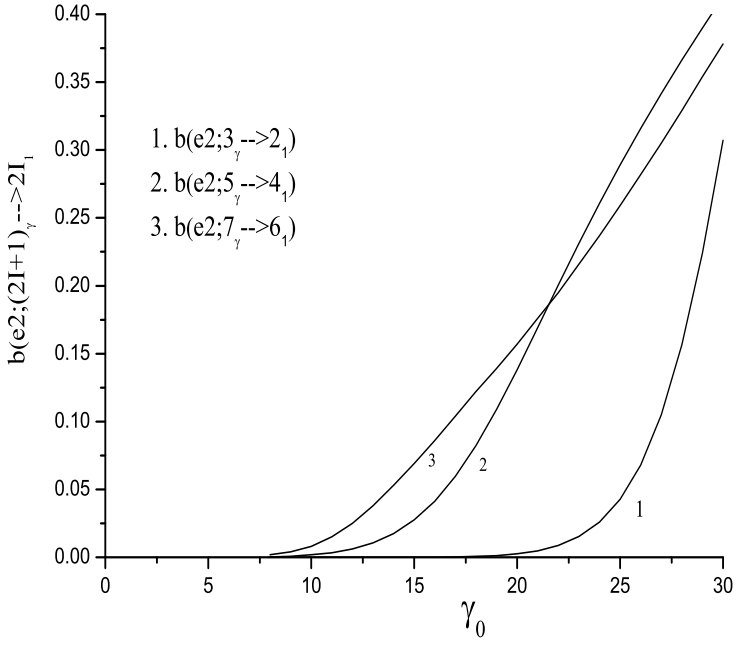


FIG. 11. The same as in Fig. 1, but for b(E2,$(2I+1)_\gamma \rightarrow 2I_1$).

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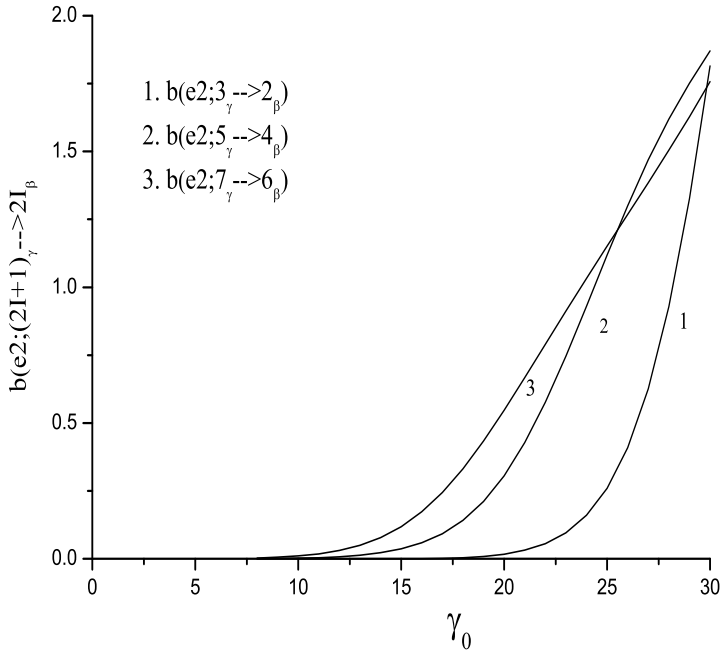


FIG. 12. The same as in Fig. 1, but for b(E2,$(2I+1)_\gamma \to 2I_\beta$).

12 behavior of the inter-band reduced $E2$-transitions probabilities between same spins $(2I+1)_\gamma \to 2I_\beta$ is presented. These transitions are observed in the collective excitation spectra of the following deformed even-even nucleus: ^{168}Er [19]. From fig. 1-12 we can see, that the complex behavior inter/intra-band reduced $E2$-transitions probabilities at large values of the γ triaxiality parameter is bandcrossing between the ground and γ-band.

III. CONCLUSION

In this paper the role of dynamic triaxality in the quadrupole shape excitation in even-even heavy nuclei is considered in the framework of the free triaxiality model, where γ and β_2 variables are dynamic. The Davidson potential is used for γ and β_2 variables. The expression for intra/inter-band reduced E2-transition probabilities ground-state-band, β- and γ-bands are being obtained and are being expressed in terms of four parameters: μ_β, μ_γ, γ_0 and Q_0. The branching ratio of intra/inter-band reduced E2-transition probabilities ground-state-band, β- and γ-bands for heavy even-even nuclei are considered. The influence of γ-oscillations to the reduced probabilities of E2-transitions takes into account are numerically. The sensitivity of the intra/inter-band reduced $E2$-transition probabilities to the triaxiality parameter γ_0 was analyzed. The regions of mass numbers of even-even nuclei where these transitions are observed are also determined.

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