Journal of Fundamental and Applied Research **Vol. 3**, Issue 2 (2023) **20230010**

Received: 26 April 2023 / Accepted: 29 May 2023
Published online: 1 June 2023
© 2023 The Author(s)

Test particle around Magnetically charged Einstein-Euler-Heisenberg black hole

Odilbek Kholmuminov[1, *] and Azamjon Boydedayev[1, †]

[1] *Ulugh Beg Astronomical Institute, Astronomy Street 33, Tashkent 100052, Uzbekistan*

In this study, the motion of neutral test particles around a magnetically charged Einstein-Euler-Heisenberg black hole is investigated. The black hole's spacetime is described by a metric that incorporates magnetic charge and a coupling parameter for Non-Linear Electrodynamics. The effective potential for test particles in this spacetime is derived, revealing its radial dependence. The results indicate that the electric charge of the black hole significantly affects the effective potential, while the coupling parameter has a more modest influence. Circular orbits are analyzed, yielding explicit expressions for the specific energy and specific angular momentum of test particles in these orbits. The specific energy is found to be primarily affected by the electric charge, with the coupling parameter playing a minor role. Similarly, the specific angular momentum exhibits behavior consistent with the specific energy. In summary, this study sheds light on the dynamics of test particles around a magnetically charged Einstein-Euler-Heisenberg black hole, highlighting the distinct impact of electric charge and the relatively minor influence of the NLED coupling parameter on particle motion within this spacetime.

Keywords: Black hole, particle dynamics, Hamilton-Jacobi equation, effective potential, circular orbits, non-linear electrodynamics

I. INTRODUCTION

In the work we aim to probe the spacetime around Einstein-Euler-Heisenberg black hole through the motion of massive particles. It is worth noting that particle dynamics can give essential information about the spacetime around compact objects (see e.g. [1–16]).

The metric describing the spacetime of a magnetically charged Einstein-Euler-Heisenberg black hole is given by the following line element. [17]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,, \tag{1}$$

$$f(r) = 1 - \frac{2M}{r} + \frac{q_m^2}{r^2} - \frac{2\mu}{5}\frac{q_m^4}{r^6} \,. \tag{2}$$

In this context, the spacetime parameters M, q_m, and μ correspond to the black hole's mass, magnetic charge, and the coupling parameter for Non-Linear Electrodynamics (NLED), respectively. It's worth mentioning that when $\mu = 0$, it reverts to the Reissner-Nordstrom solution, and when $\mu = 0$, it becomes the Schwarzschild solution.

II. PARTICLE MOTION

To explore the motion of neutral test particles around magnetically charged Einstein-Euler-Heisenberg black hole, I employ the Hamilton-Jacobi (HJ) equation of motion. This equation can be expressed as follows

$$g^{\mu\nu}\frac{\partial S}{x^\mu}\frac{\partial S}{x^\nu} = -m^2 \,, \tag{3}$$

* xolmominovodilbek308@gmail.com
† azamjon@astrin.uz

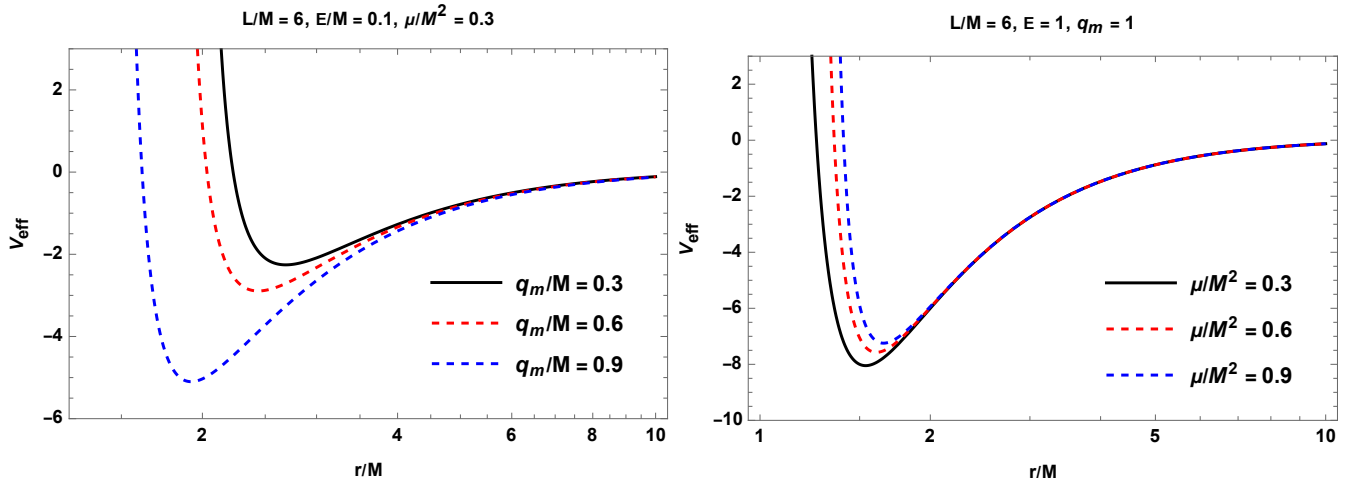


FIG. 1. The radial variation of the effective potential for a test particle in orbit around a magnetically charged Einstein-Euler-Heisenberg black hole is depicted.

Here, $g^{\mu\nu}$ represents the contravariant components of the metric tensor, S denotes the action, and m specifies the mass of the test particle. Given the spherical symmetry of the spacetime in question, the action can be expressed in the following manner.

$$S = -Et + L\phi + R(r) + T(\theta) \tag{4}$$

Considering that E and L represent the energy and angular momentum of the test particle, respectively, we can solve the equation of motion (3) while taking into account the previous expression. This leads to the effective potential of the test particle taking the following form (when $\theta = \pi/2$).

$$V_{eff} = -\frac{5\mathcal{E}^2 r^6}{5r^5(r-2M) - 2\mu q^4 + 5q^2 r^4} - \frac{\mathcal{L}^2}{r^2} - 1\,, \tag{5}$$

considering specific energy $\mathcal{E} = E/m$ and specific angular momentum $\mathcal{L} = L/m$. The radial dependence of the effective potential is presented in Fig. 1. In the left panel, it's evident that increasing the electric charge of a black hole decreases the effective potential for a fixed coupling parameter μ. Meanwhile, in the right panel, one can observe that the impact of the coupling parameter is relatively modest when compared to the electric charge, and an increase in this parameter results in higher values of the effective potential.

Now, let's examine the scenario of circular orbits, which necessitates the fulfillment of certain conditions by the effective potential, namely,

$$V_{eff} = 0\,, \tag{6}$$
$$\partial_r V_{eff} = 0\,. \tag{7}$$

These conditions allow us to determine the specific energy and specific angular momentum of a test particle following circular orbits, which can be expressed as follows

$$\mathcal{E} = \frac{-\frac{2M}{r} - \frac{2\mu q^4}{5r^6} + \frac{q^2}{r^2} + 1}{\sqrt{-\frac{3M}{r} - \frac{8\mu q^4}{5r^6} + \frac{2q^2}{r^2} + 1}} \tag{8}$$

$$\mathcal{L} = \frac{r\sqrt{\frac{5Mr^5 + 6\mu q^4 - 5q^2 r^4}{r^6}}}{\sqrt{-\frac{15M}{r} - \frac{8\mu q^4}{r^6} + \frac{10q^2}{r^2} + 5}} \tag{9}$$

The relationship between the specific energy of the test particle and the radius of circular orbits is displayed in Fig. 2. As evident in the left plot, an increase in the coupling parameter has a minimal impact, particularly at greater distances, while the electric charge exerts a considerably stronger influence, as illustrated in the right panel. The radial profile of the specific angular momentum can be readily visualized, as shown in Fig. 3. It's apparent that the specific angular momentum of the test particle exhibits a behavior akin to that of the specific energy.

 Copyright © 2023

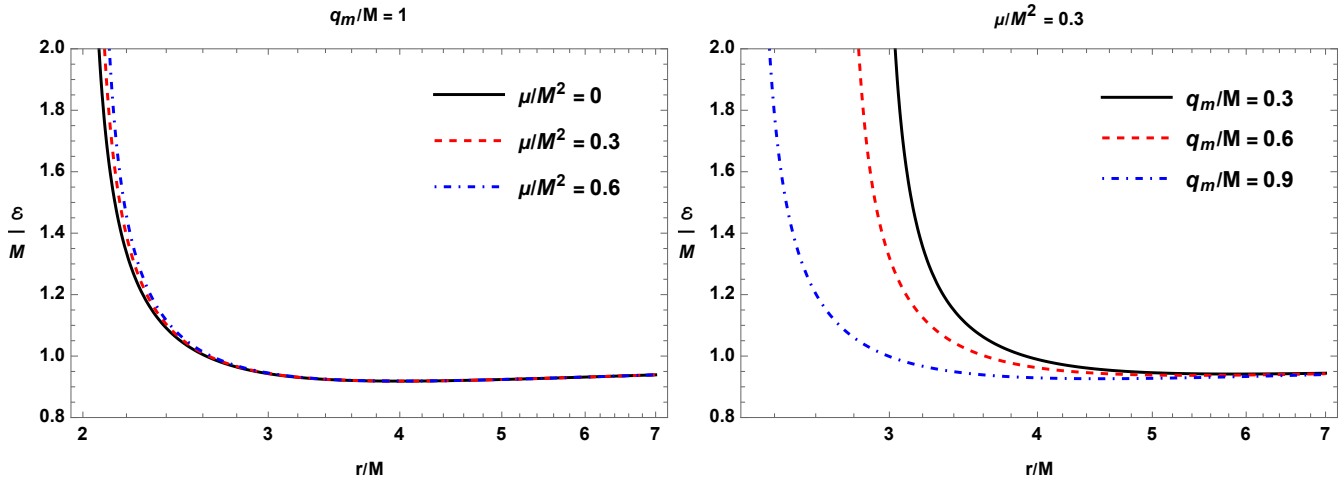


FIG. 2. Variations in the specific energy of the test particle in response to alterations in the radius of circular orbits are depicted for various values of the spacetime parameters.

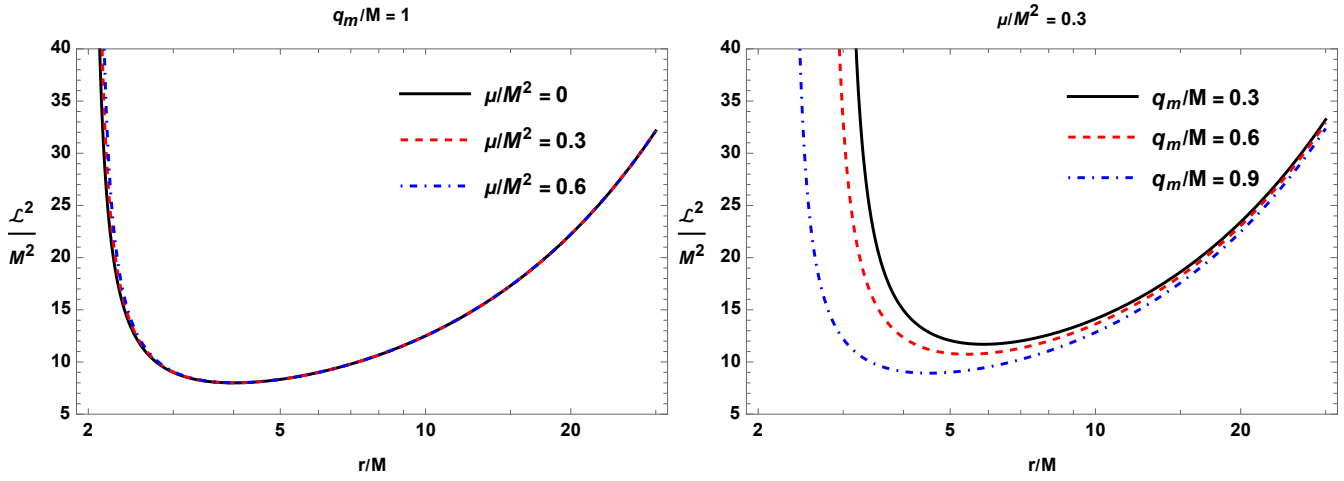


FIG. 3. The relationship between the angular momentum of a test particle moving in the equatorial plane and the radius of circular orbits is demonstrated.

CONCLUSION

In this investigation, the influence of the electric charge and NLED coupling parameter on the dynamics of test particles within the equatorial plane of a Magnetized Einstein-Euler-Heisenberg black hole was explored. The Hamilton-Jacobi equation was utilized to derive the explicit form of the test particle's effective potential, revealing its radial dependence. Notably, the effective potential increases with rising coupling parameter and decreases with growing electric charge.

Furthermore, the study delved into the circular motion of test particles within the equatorial plane, unveiling the explicit expressions for specific energy and specific angular momentum. The relationship between these quantities and the radius of circular orbits was also presented. Remarkably, the coupling parameter exhibited a negligible impact on these parameters, while the electric charge had a significant diminishing effect, particularly in lower orbits.

[1] S. Shaymatov, J. Vrba, D. Malafarina, B. Ahmedov, and Z. Stuchlík, Physics of the Dark Universe **30**, 100648 (2020).
[2] F. Atamurotov, S. Shaymatov, P. Sheoran, and S. Siwach, Journal of Cosmology and Astroparticle Physics **2021**, 045 (2021).

Copyright © 2023

[3] J. Rayimbaev, S. Shaymatov, and M. Jamil, The European Physical Journal C **81**, 1 (2021).
[4] B. Turimov, J. Rayimbaev, A. Abdujabbarov, B. Ahmedov, and Z. Stuchlík, Physical Review D **102**, 064052 (2020).
[5] B. Gao and X.-M. Deng, The European Physical Journal C **81**, 983 (2021).
[6] F. Atamurotov, S. Shaymatov, and B. Ahmedov, Galaxies **9**, 54 (2021).
[7] S. Shaymatov, P. Sheoran, and S. Siwach, Physical Review D **105**, 104059 (2022).
[8] S. Shaymatov, D. Malafarina, and B. Ahmedov, Physics of the Dark Universe **34**, 100891 (2021).
[9] J. Rayimbaev, S. Shaymatov, F. Abdulxamidov, S. Ahmedov, and D. Begmatova, Universe **9**, 135 (2023).
[10] H.-Y. Lin and X.-M. Deng, Annals of Physics , 169360 (2023).
[11] B. Narzilloev, A. Abdujabbarov, C. Bambi, and B. Ahmedov, Phys. Rev. D **99**, 104009 (2019), arXiv:1902.03414 [gr-qc].
[12] B. Narzilloev, J. Rayimbaev, S. Shaymatov, A. Abdujabbarov, B. Ahmedov, and C. Bambi, Phys. Rev. D **102**, 044013 (2020), arXiv:2007.12462 [gr-qc].
[13] B. Narzilloev, J. Rayimbaev, S. Shaymatov, A. Abdujabbarov, B. Ahmedov, and C. Bambi, Phys. Rev. D **102**, 104062 (2020), arXiv:2011.06148 [gr-qc].
[14] B. Narzilloev, D. Malafarina, A. Abdujabbarov, and C. Bambi, Eur. Phys. J. C **80**, 784 (2020), arXiv:2003.11828 [gr-qc].
[15] B. Narzilloev, J. Rayimbaev, A. Abdujabbarov, and C. Bambi, The European Physical Journal C **80**, 1074 (2020), arXiv:2005.04752 [gr-qc].
[16] B. Narzilloev, J. Rayimbaev, A. Abdujabbarov, B. Ahmedov, and C. Bambi, Eur. Phys. J. C **81**, 269 (2021), arXiv:2103.11090 [gr-qc].
[17] A. Allahyari, M. Khodadi, S. Vagnozzi, and D. F. Mota, JCAP **02**, 003 (2020), arXiv:1912.08231 [gr-qc].

Copyright © 2023