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Effect of magnetized plasma on an observable shadow of a Reissner-Nordström black hole

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In this paper, we study the influence of the axion-plasmon, as proposed in (Phys. Rev. Lett. 120, 181803 (2018)) on the shadow of the charged black hole. We only consider spherically symmetric and static situations, where the shadow is circular. Our main aim is to plot dependence of the shadow radius on different parameters including black hole's charge Q, plasma frequency ω_p, axion frequency ω_ϕ, homogeneous magnetic field B_0 . To accomplish our goal, we investigate the effect of the axion-plasmon coupling on the motion of photons around the Reissner–Nordström black hole. Moreover, we plot the graphs of photon sphere depending on axion-plasmon parameters and a black hole's charge. We explore the effects of homogeneous plasma with $\omega_p^2 = const$, and inhomogeneous plasma with power-law model of plasma frequency $\omega_p^2 = z_0/r^q$ on the black hole shadow. We find that for an observer far away from the Reissner–Nordström black hole with increasing axion-plasmon, the black hole shadow decreases.

Key words: Reissner–Nordström black hole, shadow, axion, photon sphere, photon motion, homogeneous and inhomogeneous

I. INTRODUCTION

Black holes are remarkable predictions of Einstein's theory of general relativity with significant implications. They play a central role in various high-energy astronomical phenomena and contribute to our understanding of space, time, and quantum gravity [1, 2] . Current research on black holes focuses on addressing information loss paradox, holography, and entanglement. The event horizon is a key feature of a black hole, preventing light from escaping and making it unobservable. However, the spacetime geometry near the event horizon can be probed through photon rings and the black hole shadow [3–5]. In recent years, the Event Horizon Telescope closely observed the shadows of a supermassive black hole at the center of the M87 galaxy and later the one at the heart of our Milky Way galaxy, known as Sgr A* [6]. Through these observations, astronomers discovered significant magnetic field structures, plasma accretion, and determined the accretion rate of the central black hole in M87 [7]. The presence of the dark shadow confirms the existence of a supermassive black hole at the galactic center and provides further support for the theory of general relativity. Researchers have also studied modified gravity and quantum gravity-inspired black holes to analyze their shadows and impose constraints on their properties using observational data [8–15]. In this context, we investigate the influence of plasma and magnetic fields on the shadow [16] of an electrically charged black hole using the Reissner-Nordström BH model. The paper is divided into sections:Sec. II examines photon motion around a magnetized plasma near a black hole, while Sec. III focuses on the axion-plasmon effect on the black hole's shadow in more detail. Finally, the results are discussed in Sec. IV. Throughout the paper, we use a system of geometric units in which $G = 1 = c$. Greek indices run from 0 to 3.

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II. PHOTON MOTION AROUND THE BH IN THE PRESENCE OF AXION-PLASMON

We consider a generalized electromagnetic theory taking into account the axion-photon coupling [17, 18]

$$\mathcal{L} = R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_\mu J_e^\mu + \mathcal{L}_\varphi + \mathcal{L}_{\rm int}, \tag{1}$$

where R, $F_{\mu\nu}$ and J_e^μ denote the Ricci scalar, electromagnetic tensor and the four vector current of electrons respectively while $\mathcal{L}_\varphi = \nabla_\mu\varphi^*\nabla^\mu\varphi - m_\varphi^2|\varphi|^2$, is the axion Lagrangian density, finally $\mathcal{L}_{\rm int} = -(g/4)\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}F_{\mu\nu}$, is the photon-axion interaction term where g denotes the relevant coupling.

The spacetime metric of the Reissner–Nordström BH which is a static and spherically symmetric is given by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

here $f(r) = 1 - (2M/r) + (Q^2/r^2)$ and M and Q denote mass and charge of the BH, respectively.

The Hamiltonian of a photon orbiting around a BH surrounded by an axion-plasmon medium has the following form [19]

$$\mathcal{H}(x^\alpha, p_\alpha) = \frac{1}{2}\left[g^{\alpha\beta}p_\alpha p_\beta - (n^2-1)(p_\beta u^\beta)^2\right], \tag{3}$$

where x^α are the spacetime coordinates, p_α and u^β are the four-momentum and four-velocity of the photon respectively and n is the refractive index ($n = \omega/k$, where k is the wave number). In the case of an axion-plasmon contribution, the refractive index is expressed as follows [17]

$$\begin{aligned} n^2 &= 1 - \frac{\omega_{\rm p}^2}{\omega^2} - \frac{f_0}{\gamma_0}\frac{\omega_{\rm p}^2}{(\omega - ku_0)^2} - \frac{\Omega^4}{\omega^2(\omega^2-\omega_\varphi^2)} \\ &\quad - \frac{f_0}{\gamma_0}\frac{\Omega^4}{(\omega-ku_0)^2(\omega^2-\omega_\varphi^2)}, \end{aligned} \tag{4}$$

in terms of the plasma frequency $\omega_p^2(x^\alpha) = 4\pi e^2 N(x^\alpha)/m_e$ (e and m_e are the electron charge and mass respectively whereas N is the number density of the electrons), the photon frequency $\omega(x^\alpha)$ is defined by $\omega^2 = (p_\beta u^\beta)^2$, the axion frequency ω_φ^2, the axion-plasmon coupling parameter $\Omega = (gB_0\omega_p)^{1/2}$ with B_0 being the homogeneous magnetic field in the z-direction. The parameter f_0 is the fraction of the electrons in the beam propagating inside the plasma with velocity u_0 and γ_0 is the corresponding Lorentz factor. Because the role of the electron beam scenario near the BH is less clear, we set $f_0 = 0$ for simplicity and rewrite (4) as

$$\begin{aligned} n^2(r) &= 1 - \frac{\omega_{\rm p}^2(r)}{\omega(r)^2} - \frac{\Omega^4}{\omega(r)^2[\omega(r)^2-\omega_\varphi^2]}, \\ &= 1 - \frac{\omega_{\rm p}^2(r)}{\omega(r)^2}\left(1 + \frac{g^2B_0^2}{\omega(r)^2-\omega_\varphi^2}\right), \end{aligned} \tag{5}$$

with

$$\omega(r) = \frac{\omega_0}{\sqrt{f(r)}}, \qquad \omega_0 = \text{const.} \tag{6}$$

Experiments concerning the axion-plasmon conversion impose the following constraint upon frequency scales $\omega_{\rm p}^2 \gg \Omega^2$ or $\omega_{\rm p} \gg gB_0$ [17]. The lapse function is such that $f(r) \to 1$ as $r \to \infty$ and $\omega(\infty) = \omega_0 = -p_t$, which represents energy of the photon at spatial infinity [20]. Besides, the plasma frequency must be sufficiently small than the photon frequency ($\omega_{\rm p}^2 \ll \omega^2$) which allows the BH shadow to be differentiated from the vacuum case. The Hamiltonian for the light rays in the axion-plasmon medium has the form

$$\mathcal{H} = \frac{1}{2}\Big[g^{\alpha\beta}p_\alpha p_\beta + \omega_{\rm p}^2\Big(1 + \frac{g^2B_0^2}{\omega_0^2-\omega_\varphi^2}\Big)\Big]. \tag{7}$$

The components of the four velocity for the photons in the equatorial plane ($\theta = \pi/2,\ p_\theta = 0$) are given by

$$\dot{t} \equiv \frac{dt}{d\lambda} = \frac{-p_t}{f(r)}, \tag{8}$$

$$\dot{r} \equiv \frac{dr}{d\lambda} = p_r f(r), \tag{9}$$

$$\dot{\phi} \equiv \frac{d\phi}{d\lambda} = \frac{p_\phi}{r^2}, \tag{10}$$

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where we used the relationship, $\dot{x}^\alpha = \partial\mathcal{H}/\partial p_\alpha$. From Eqs. (9) and (10), we obtain a governing equation for the phase trajectory of light

$$\frac{dr}{d\phi} = \frac{g^{rr}p_r}{g^{\phi\phi}p_\phi}. \tag{11}$$

Using the constraint $\mathcal{H} = 0$, we can rewrite the above equation as [20]

$$\frac{dr}{d\phi} = \sqrt{\frac{g^{rr}}{g^{\phi\phi}}}\sqrt{h^2(r)\frac{\omega_0^2}{p_\phi^2} - 1}, \tag{12}$$

where we defined

$$h^2(r) \equiv -\frac{g^{tt}}{g^{\phi\phi}} - \frac{\omega_p^2}{g^{\phi\phi}\omega_0^2}\left(1 + \frac{g^2B_0^2}{\omega_0^2 - \omega_\varphi^2}\right). \tag{13}$$

We now introduce the dimensionless parameters

$$\tilde{\omega}_\varphi^2 = \frac{\omega_\varphi^2}{\omega_0^2} \quad \text{and} \quad \tilde{B}^2 = \frac{g^2B_0^2}{\omega_0^2}, \tag{14}$$

which yield

$$h^2(r) = r^2\Big[\frac{r^2}{r^2 - 2Mr + Q^2} - \frac{\omega_{\rm p}^2(r)}{\omega_0^2}\Big(1 + \frac{\tilde{B}^2}{1 - \tilde{\omega}_\varphi^2}\Big)\Big]. \tag{15}$$

The radius of a circular orbit of light, particularly the one which forms the photon sphere of radius $r_{\rm ph}$, is determined by solving the following equation [20]

$$\frac{d(h^2(r))}{dr}\bigg|_{r=r_{\rm ph}} = 0. \tag{16}$$

By substituting Eq. (15) into (16) one can write the algebraic equation for $r_{\rm ph}$ in the presence of plasma medium as

$$\begin{aligned}\Big[\frac{\omega_{\rm p}^2(r_{\rm ph})}{\omega_0^2} + \frac{r_{\rm ph}\omega_{\rm p}'(r_{\rm ph})\omega_{\rm p}(r_{\rm ph})}{\omega_0^2}\Big]\Big(1 + \frac{\tilde{B}_0^2}{1-\tilde{\omega}_\varphi^2}\Big) = \\ = \frac{r_{\rm ph}^4 - 3r_{\rm ph}^3M + 2Q^2r_{\rm ph}^2}{(r_{\rm ph}^2 - 2Mr_{\rm ph} + Q^2)^2},\end{aligned} \tag{17}$$

where prime denotes the derivative with respect to radial coordinate r. Although the roots of Eq. (17) cannot be obtained analytically for most choices of $\omega_{\rm p}(r)$, we can plot graphs for the specific options of $\omega_{\rm p}(r)$ numerically.

A. Homogeneous plasma with $\omega_p^2(r) = $ const.

In the special case of a homogeneous plasma with axions i.e. $\omega_p^2 = $ const., the exact form of the photon sphere cannot be obtained, because of this we plot the graph by sitting some values for $\tilde{\omega}_\phi^2, \omega_p^2/\omega_0^2, Q^2$ and $\tilde{B}^2$. Plots of $r_{\rm p}$ are depicted in Fig. 1 versus the plasma frequency, the magnetic field, the axion frequency and the charge of the black hole seperately. The figures suggest that while the first three physical factors contribute to increase the size of the photon sphere, the charge effects conversely.

B. Inhomogeneous plasma with $\omega_p^2(r) = z_0/r^q$

Now we explore photon spheres in the presence of an inhomogeneous plasma with axion, where the plasma frequency is required to satisfy a simple power-law of the form [21, 22]

$$\omega_p^2(r) = \frac{z_0}{r^q}, \tag{18}$$

where z_0 and q are free parameters. To analyze the main features of the power-law model we restrict ourselves to the case $q = 1$ and z_0 as a constant [21]. Again we have plotted graphs numerically and chose the solution of [17] when there is no plasma contribution ($z_0 = 0$), then the root reduces to $r_{\rm ph} = \frac{1}{2}\big(3M + \sqrt{9M^2 - 8Q^2}\big)$, which is the photon sphere for a non-rotating charged BH. In Fig. 2 the radius of photon sphere versus different free parameters are given. The effects of parameters of axion-plasmon model on the size of photon sphere are evidently manifested.

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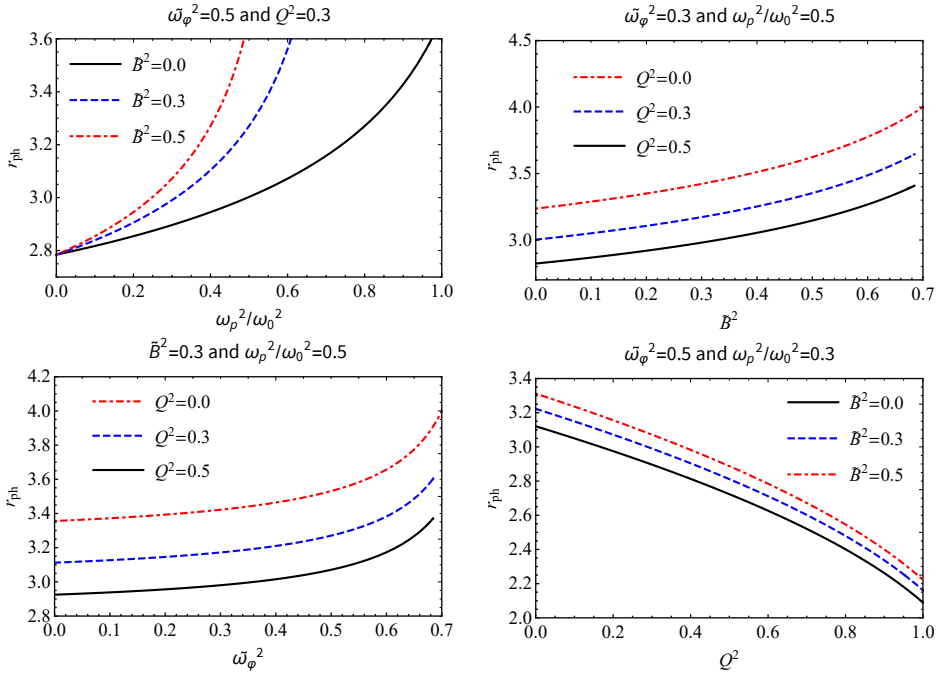


FIG. 1. Radius of the photon sphere for the homogeneous plasma with axion field.

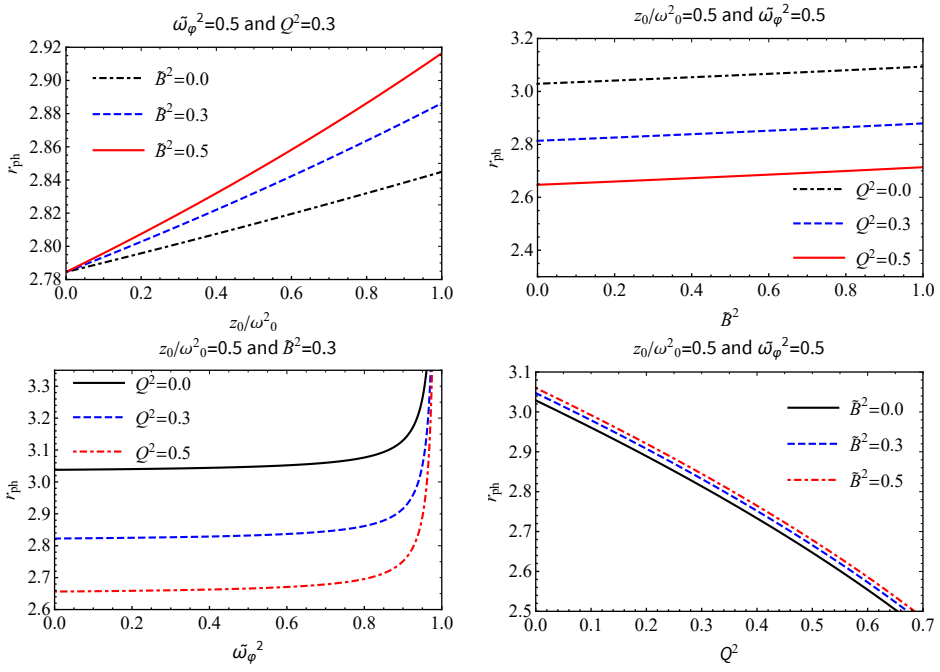


FIG. 2. Radius of photon sphere for the inhomogeneous power-law plasma with axion field.

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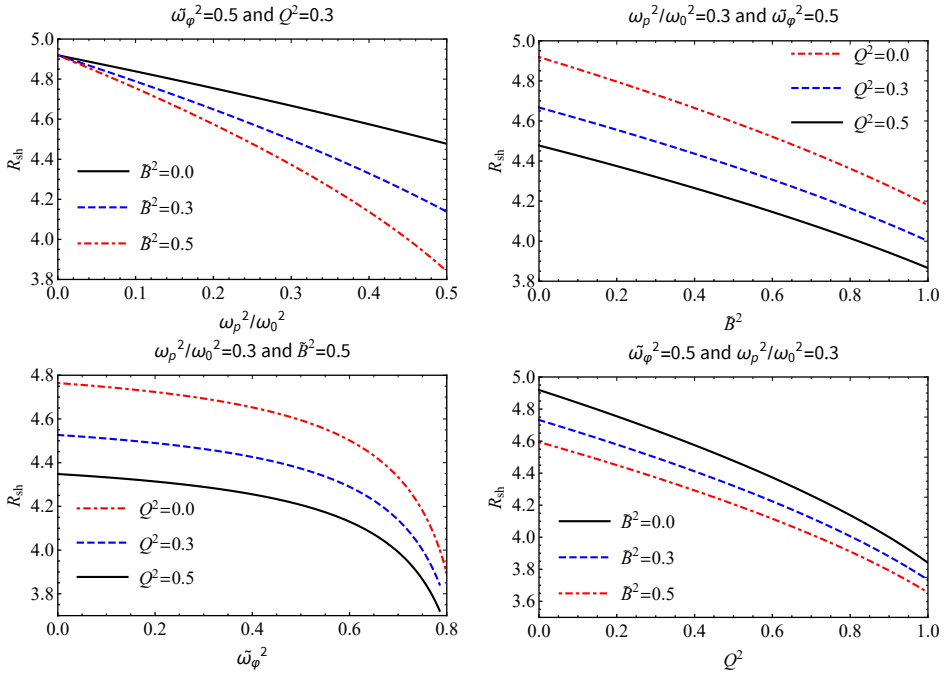


FIG. 3. Shadow's radius of the BH for the homogeneous constant-frequency plasma with axion field

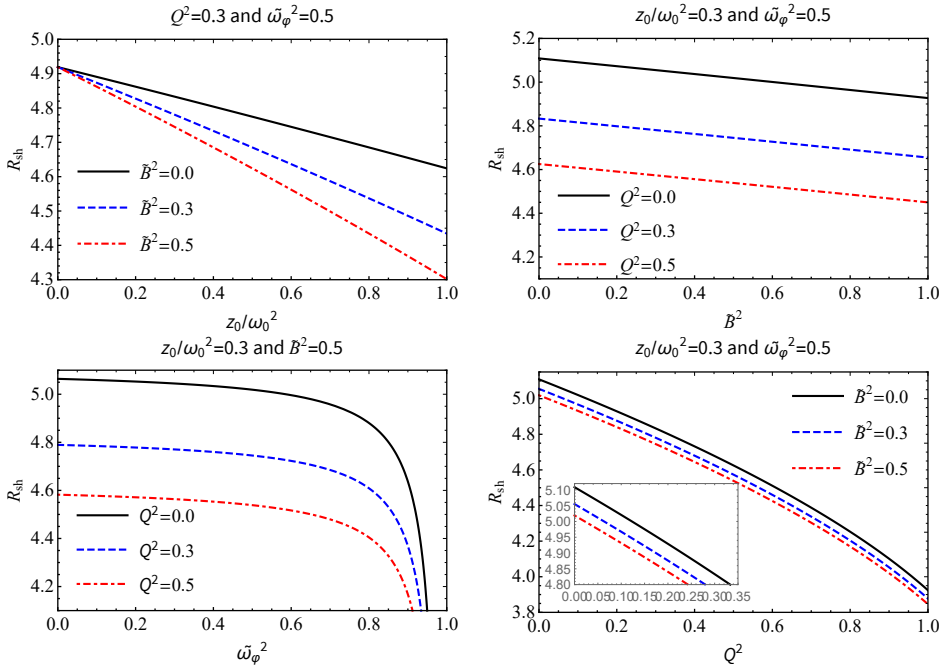


FIG. 4. Radius of the BH shadow for the inhomogeneous plasma with axion field and power-law plasma frequency.

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III. BH SHADOW IN AN AXION-PLASMON MEDIUM

In this section we study the radius of the shadow of the Reissner–Nordström metric in the presence of a magnetized plasma. The angular radius $\alpha_{\rm sh}$ of the BH shadow is defined by a geometric approach which results in [20, 23]

$$\sin^2\alpha_{\rm sh} = \frac{h^2(r_{\rm ph})}{h^2(r_{\rm o})} = \\ = \frac{r_{\rm ph}^2\left[\frac{r_{\rm ph}^2}{r_{\rm ph}^2-2Mr_{\rm ph}+Q^2}-\frac{\omega_p^2(r_{\rm ph})}{\omega_0^2}\left(1+\frac{\tilde{B}^2}{1-\tilde{\omega}_\varphi^2}\right)\right]}{r_{\rm o}^2\left[\frac{r_{\rm o}^2}{r_{\rm o}^2-2Mr_{\rm o}+Q^2}-\frac{\omega_p^2(r_{\rm o})}{\omega_0^2}\left(1+\frac{\tilde{B}^2}{1-\tilde{\omega}_\varphi^2}\right)\right]}, \tag{19}$$

where $r_{\rm o}$ and $r_{\rm ph}$ represent the locations of the observer and the photon sphere respectively. If the observer is located at a sufficiently large distance from the BH then one can approximate radius of BH shadow by using Eq. (19) as [20]

$$R_{\rm sh} \simeq r_{\rm o}\sin\alpha_{\rm sh} = \\ = \sqrt{r_{\rm ph}^2\left[\frac{r_{\rm ph}^2}{r_{\rm ph}^2-2Mr_{\rm ph}+Q^2}-\frac{\omega_p^2(r_{\rm ph})}{\omega_0^2}\left(1+\frac{\tilde{B}^2}{1-\tilde{\omega}_\varphi^2}\right)\right]}, \tag{20}$$

where we have used the fact that $h(r) \to r$, which follows from Eq. (15), at spatial infinity for both models of plasma along with a constant magnetic field. In the case of vacuum $\omega_{\rm p}(r) \equiv 0$, and considering Q=0 we recover the radius of Schwarzschild BH shadow $R_{\rm sh} = 3\sqrt{3}M$ when $r_{\rm ph} = 3M$. The radius of BH shadow is depicted for different parameters in Fig. 3 for a homogeneous plasma with fixed plasma frequency and Fig. 4 shows the case for a power-law model of plasma frequency $\omega_p^2(r) = z_0/r$. We observe that the size of shadow radius decreases by increasing the magnetic field strength or the axion-plasmon frequency. Thus the BH shadow in the presence of axion-plasmon medium would shrink further, as expected. It is interesting to note that the effects of a homogeneous plasma on the radius of the BH shadow are more pronounced than the effects of an inhomogeneous plasma, which is abundantly clear from Figs. 3 and Figs. 4.

IV. CONCLUSIONS

In this study, we examined how the interaction between axions and photons affect the optical properties of a Reissner–Nordström black hole (BH). Specifically, we focused on observing the BH shadow. We found that the size of the BH shadow decreases as the axion-plasmon effect increases, especially when observing from a large distance. This phenomenon had previously been observed in the case of an inhomogeneous plasma. The size of the shadow may be larger for a closer observer, but if the observer is far from the BH, the shape of the BH's shadow will be smaller and influenced by the type of plasma present. Our findings suggest that when one of the three parameters is varied while holding the other two constant, the effects of a homogeneous plasma on the radius of the photon sphere and the shadow of the BH are more significant compared to an inhomogeneous plasma.

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