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# Thermonuclear reactions in stars

Saidmuhammad Ahmedov[1, *]

[1]*National University of Uzbekistan, Tashkent, Uzbekistan*

In this article, I will present the main fundamental ideas about the fusion reactions that take place in stars. Let's take a closer look at some of the important quantum-mechanical phenomena that govern thermonuclear reactions, and continue with how nuclear reactions proceed in hot stellar plasma. This lecture is intended for students of nuclear astrophysics. My goal is to present the unrelated phenomena of nuclear physics and astrophysics in a coherent manner.

## I. INTRODUCTION

The field of nuclear astrophysics started with basic questions regarding our Sun. It is obvious that life on Earth depends on nuclear processes deep inside the Sun, but how exactly the nuclear transmutations occur was not understood for some time. The breakthroughs came at the end of the 1930's: Bethe and Critchfield [1] uncovered a sequential reaction sequence fusing hydrogen (H) to helium (He), now referred to as the "pp1 chain", while Bethe [2] and von Weizsäcker [3] proposed a cyclic reaction sequence, now called the "CNO1 cycle", that has the same end result of synthesizing He from H. For this early work, the Nobel prize was awarded to Hans Bethe in 1967. It is interesting to point out that Bethe originally thought that the Sun derives most of its energy via the CNO1 cycle. Part of the problem was that some of the key nuclear reaction cross sections were poorly known. When more reliable cross sections could be estimated in the 1950's, it became apparent that it is in fact the pp1 chain that governs the energy production in the Sun. The important lesson is that accurate nuclear physics information is crucial for our understanding of stars.

## II. NUCLEAR REACTIONS

The cross section of a nuclear reaction is defined as the number of interactions per time, divided by the number of incident particles per area and time, and divided by the number of target nuclei within the beam. The unit is barn, where 1 $barn = 10^{-28}m^2$. For example, the estimated cross section for the reaction p + p $\rightarrow$ d + $e^+$+$\nu$, which represents the first step in the pp chains (see below), amounts to $\sigma = 8 * 10^{-48}sm^2$ at a laboratory bombarding energy of 1 MeV. Suppose a measurement of this reaction would be performed using an intense 1 mA beam of protons, incident on a dense hydrogen target ($10^{20}$ protons per $sm^2$), then one obtains only 1 interaction in 6000 years! Clearly, such a measurement is beyond present experimental capabilities and hence this cross section needs to be estimated theoretically.

These questions can best be answered by considering a simple potential model for a nuclear reaction. We need essentially two pieces: an attractive (negative) square well potential of depth $V_0$ inside the nucleus (r ¡ $R_0$), and a repulsive (positive) square barrier potential of height $V_1$ for distances of $R_0 \leq r < R_1$. The total energy of the incident particle, E, is less than the barrier height. An example for such a potential is shown in Fig. 1. The solution to the Schrodinger equation for each of the three regions I, II, III are well know and can be found in any introductory quantum mechanics textbook. In regions I and III, the solutions are in the form of complex exponentials, which represent a sine function. In region II, however, the solutions are given in terms of real exponentials. In the next step, the continuity condition is applied, that is, the wave function solutions and their derivatives must be continuous at the two boundaries $R_0$ and $R_1$. We obtain four equations and can solve for the intensity of the transmitted wave in region I, normalized to the intensity of the incident wave in region III. For relatively low energies one finds for this ratio, also called transmission coefficient, after some algebra

* saidmuhammadaxmed@gmail.com

$$T = exp(-2/h)\sqrt{2m(V_1 - E)}(R_1 - R_0) \tag{1}$$

where m denotes the reduced mass. This result, referred to as the Tunnel effect, is remarkable and represents one of the most important quantum mechanical phenomena for charged particle reactions: although the incoming particle is classically not allowed to reach region I, there is in fact a finite probability for tunneling through the barrier. Without this circumstance the world would be a very different place and life on Earth would certainly not exist. It is clear from Eq. (1) that the transmission coefficient depends very sensitively on the properties of the barrier.

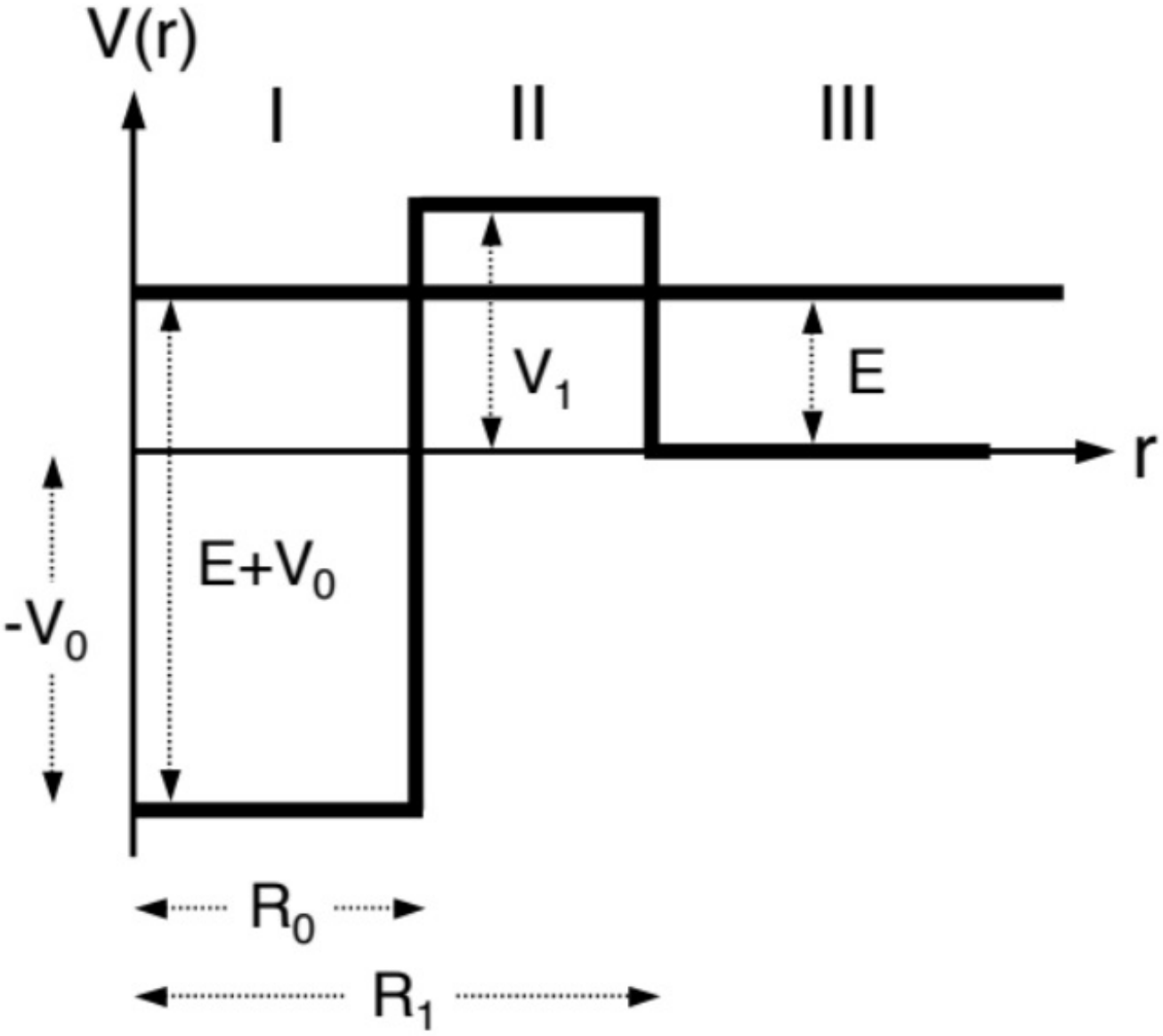



FIG. 1. Three-dimensional square-well-plus-square-barrier potential, representing the simplest potential model for a charged-particle nuclear reaction. The potential properties are defined in the text. Note that for real potential the transmission probability is only defined for the one-dimensional case. Thus for the calculation of the transmission coefficient we will assume that the figure represents a one-dimensional potential that extends to -$\infty$.

We have not considered yet the full radial wave function solution of the Schrodinger equation for the three-dimensional case. In particular, we are interested in the ratio, R, of the wave function intensities in regions I and III. It is now of advantage to express the wave function solutions in these regions as sine functions instead of complex exponentials. Otherwise we proceed as before: we apply the continuity condition and solve the system of four equations for the ratio R. After some tedious algebra

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we find a rather lengthy analytical expression. Interestingly, when plotting this function versus energy E for certain values of the potential depth $V_0$, a well-defined maximum is produced, while for other values of $V_0$ the resulting function closely reproduces the structureless energy-dependence of the transmission coefficient in Eq. (1). Note that by changing the potential depth $V_0$ we are changing the wavelength in the nuclear interior (region I), and for discrete values of $V_0$ the wave function amplitude in the nuclear interior becomes relatively large. This describes, in the simplest case, the origin of a well-defined cross section maximum: a resonance results from favorable wave function matching conditions at the nuclear boundary. In formal reaction theory, a simple equation describing a single isolated resonance can be derived. It is referred to as Breit-Wigner formula and is given by

$$\sigma_{\mathrm{BW}}(E) = \frac{\lambda^2 \omega \Gamma_{\mathrm{a}} \Gamma_{\mathrm{b}}}{4\pi (E_{\mathrm{r}} - E)^2 + \Gamma^2/4} \tag{2}$$

where $\lambda$ is the de Broglie wavelength, $\omega$ is a factor containing angular momenta, Er is the resonance energy, $\Gamma_{\mathrm{i}}$ are the resonance partial widths of entrance and exit channel, and $\Gamma$ is the total resonance width given by the sum of all partial widths. The above equation is the single most important expression describing a resonance and it is frequently used in nuclear astrophysics in many applications: (i) for fitting cross section data to extract resonance parameters; (ii) for deriving the "narrow-resonance reaction rate" (see below); (iii) for extrapolating cross sections to energy regions were no measurements exist; and (iv) for calculating the experimental resonance yield when the resonance cannot be resolved experimentally.

## III. THERMONUCLEAR REACTIONS

So, the interaction is called thermonuclear reaction. For the reaction 0+1 $\rightarrow$ 2+3, it is expressed by the rate of fusion reaction (the number of reactions per unit time and unit volume), where $N_{\mathrm{i}}$- is the number density of interacting nuclei and $(\sigma v)_{01}$- is the rate of reaction by particle pair, this rate, is equal to the integral over the product of the cross section and the velocity probability density. In most cases of practical interest, the latter function is given by the Maxwell-Boltzmann distribution.

$$(\sigma v)_{01} = (\frac{8}{\pi m_{01}})^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\sigma(E) e^{-E/kT}\, dE \tag{3}$$

Thus, the reaction rate per particle pair can be written, where $m_{01}$ is the reduced mass, k is the Boltzmann constant, and T is the plasma temperature. Obviously, the reaction rate for a given temperature is determined if the nuclear reaction cross section s (E) is known.

At this point, it should be noted that a certain nuclear reaction occurring in the stellar plasma can rarely be considered as an isolated interaction. For example, consider the high-temperature type $25°Al$. It can be destroyed in different ways: - by fission to $25°Mg$, by trapping a proton to $26°Si$, by photodissociation to $24°Mg$, etc. On the other hand, $25°Al$ is formed by - decay of $25°Si$, $24°Mg$ proton capture, photodecomposition of $26°Si$, etc. The abundance evolution of $25°Al$ during nucleosynthesis is then given by a differential equation that takes into account all mechanisms of destruction and production. Of course, such a differential equation must be written for all species involved in nuclear combustion. Thus, we end up with a system of coupled differential equations called the nuclear reaction network. A good description of how to solve this system numerically can be found in Arnett's book [1].

It is interesting to study (2) in more detail by considering a few extreme examples. We start with the simplest case, that is, an almost constant S-factor $S_0$. This case is usually called "off-resonance", but this leads to serious misunderstandings, since the formalism also applies to slowly varying resonance "tails", as will be discussed below. Substitution of the definition of the S-factor into (2) immediately shows that the reaction rate depends on the integral over the product of the Gamow and Boltzmann factors, in addition to the magnitude of $S_0$ $e^{2\pi\eta} e^{-E/kT}$. This result is very important, because it shows that where the cross section is large, the star does not burn at high energies (because the number of particles with such energies is very small); where the number of particles is maximum, the star does not burn even at very low energies (because the cross section is very small). More precisely, most nuclear reactions in plasma occur at energies where the $e^{2\pi\eta} e^{-E/kT}$ function is maximal. This well-defined energy window is called the Gamow peak.

Next, we will consider "narrow resonance". Several different definitions for narrow resonance can be found in the literature, but none of them are without problems. For simplicity, let us assume that a narrow resonance implies constant partial widths over the total width of the resonance. Substituting Eq. (1) to (2) immediately produces $(\sigma v) = [(2\pi)/(mkT)]^{3/2} h^2 e^{-E_{\mathrm{r}}/kT} \omega\gamma$. The product $\omega\gamma = G_{\mathrm{a}} G_{\mathrm{b}}/G$is proportional to the area under the narrow resonant cross-section curve and is therefore called the resonance power (in energy units). Note that the resonance energy enters exponentially in the reaction rate expression above. It must be determined very precisely, otherwise the uncertainty of the reaction rate will be relatively large.

In most cases, the energy dependence of the partial widths of the total resonance width cannot be neglected. Such "broad resonance" reaction rates should be treated with caution. Although approximate expressions exist in the literature, it is safer

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to substitute (1) for (2) and evaluate the integral numerically. Depending on the location of the broad resonance relative to the Gamow peak, there are two contributions to the overall reaction rate [4]. First, the contribution is calculated only from the "narrow resonance" reaction rate originating from the region close to the resonance energy (as seen by the factor of $e^{-E_{\rm r}/kT}$). Secondly, from the smooth variable tail of the resonance. If the broad resonance lies outside the Gamow peak, in many cases the resonance tail contributes much more than is calculated from the narrow resonance expression. Plotting such reaction rates against temperature often reveals a "twist" because the narrow resonance and broad resonance reaction rates depend on different temperatures.

## IV. CONCLUSION

In general, to estimate the overall rate of a single reaction, many different contributions must be taken into account: narrow and broad resonances, non-resonant processes, resonances below the threshold, cross-section continuity, interferences between different amplitudes, etc. Each reaction represents a unique situation and the evaluation process is usually tedious. Reaction rate estimates have been presented for many years by W. Fowler and his collaborators, and their last estimate (covering the target mass range A=1-30) was published in 1988 [2]. Another evaluation involving radioactive target nuclei for the first time was published in 2001 [4]. In this work, we developed a new large-scale version of reaction rate estimation using Monte Carlo methods [5].

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