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Particle acceleration by electromagnetic wave in the presence of radiation reaction force

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The effects of the radiation reaction force on charged particle motion in the presence of the external electromagnetic wave has been studied. It is shown that equation of motion reduces to the simple form under the assumption that the highest order of velocity of charged particle is negligibly small in comparison with other terms. The exact analytical solution for particle position has been found. It is shown that because of the damping parameter oscillation of particle is not longer lived.

A charged particle motion in the external electromagnetic field is quite interesting and important topic in many different areas of physics and astrophysics, in particular, after observation of astrophysical black holes evidence, such as images of supermassive black hole [1–6], X-ray flare in the close vicinity of supermassive black hole [7–9] and stellar X-ray binary sources contains the black hole or neutron star together with companion stars. Considering charged particles in the external electromagnetic field can provide information about the energetic process near such gravitational compact objects, namely, black holes, neutron stars and their electromagnetic properties that are regarded rich observational phenomenology in high-energy electromagnetic radiation.

The acceleration of charged particle in the electromagnetic wave has been studied in [10, 11]. The radiation reaction force on a point charge accelerating parallel to its velocity has been studied in [12–20]. The general relativistic effect on charged particle including the radiation reaction force has been studied in [21, 22].

In this paper we are interested in considering charged particle in the external electromagnetic wave including radiation reaction force. The relativistic equations of motion for particle of mass m and charge q in the presence of the external electromagnetic fields $\mathbf{E}$, $\mathbf{B}$ is given as [23]

$$\frac{d}{dt}\left(\gamma\mathbf{v}\right)=\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)+\tau\frac{d}{dt}\left[\gamma\frac{d}{dt}\left(\gamma\mathbf{v}\right)\right]\,, \tag{1}$$

where $\mathbf{v}=\dot{\mathbf{r}}=d\mathbf{r}/dt$ is the spatial velocity of charged particle, $\gamma=1/\sqrt{1-v^2/c^2}$ is the Lorentz factor with $v=|\mathbf{v}|$, and $\tau=2e^2/(3mc^3)$ is the radiation damping time. The last term of above equation is responsible for a radiation reaction force which is small then other terms and absence of damping time equation (1) reduces to the Lorentz equation. Hereafter applying Landau approximation up to the first power of damping time, the equation (1) reduces to

$$\frac{d}{dt}\left(\gamma\mathbf{v}\right)=\frac{e}{m}\left(1+\tau\frac{d}{dt}\gamma\right)\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\,, \tag{2}$$

while using the time variation of energy of relativistic particle one gets

$$\frac{d}{dt}\gamma=\frac{e}{mc^2}\left(1+\tau\frac{d}{dt}\gamma\right)\left(\mathbf{v}\cdot\mathbf{E}\right)\,. \tag{3}$$

The next leading order effect from damping can be obtained from the following expression:

$$\frac{d}{dt}\left(\gamma\mathbf{v}\right)=\frac{e}{m}\left(1+\tau\frac{d}{dt}\gamma+\tau^2\frac{d}{dt}\gamma\frac{d}{dt}\gamma+...+\tau^n\frac{d}{dt}\gamma...\frac{d}{dt}\gamma\right)\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\,. \tag{4}$$

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Here we focus on charged particle acceleration by plane wave in linear approximation of the radiation reaction. For simplicity, let's consider the components of the circularly polarized electromagnetic wave in the following particular form:

$$E_x = B_y = E_0 \cos(\omega t - kz)\,, \quad E_y = -B_x = E_0 \sin(\omega t - kz)\,, \quad E_z = B_z = 0\,, \tag{5}$$

which represents plane wave propagating along the z-direction, where $k = \omega/c$ is wave vector, ω is the angular frequency and E_0 is the amplitude of the electromagnetic wave. Then the components of the equation of motion yield

$$\frac{d}{dt}(\gamma v_x) = \frac{e}{m}\left(1 + \tau\frac{d}{dt}\gamma\right)\left(E_x - \frac{1}{c}v_z B_y\right)\,, \tag{6}$$

$$\frac{d}{dt}(\gamma v_y) = \frac{e}{m}\left(1 + \tau\frac{d}{dt}\gamma\right)\left(E_y + \frac{1}{c}v_z B_x\right)\,, \tag{7}$$

$$\frac{d}{dt}(\gamma v_z) = \frac{e}{mc}\left(1 + \tau\frac{d}{dt}\gamma\right)(v_x B_y - v_y B_x)\,, \tag{8}$$

$$\frac{d}{dt}(\gamma c) = \frac{e}{mc}\left(1 + \tau\frac{d}{dt}\gamma\right)(v_x E_x + v_y E_y)\,. \tag{9}$$

Our main target is to present an exact analytical solutions of the above equations. Using explicit expressions of the electromagnetic field in (5) and recalling definition of the spatial velocity $v_x = \dot{x}$, $v_y = \dot{y}$, and $v_z = \dot{z}$, equations (6)-(9) can be rewritten as

$$\frac{d}{dt}(\gamma\dot{x}) = \frac{eE_0}{m}\left(1 + \tau\frac{d}{dt}\gamma\right)\left[\left(1 - \frac{\dot{z}}{c}\right)\cos(\omega t - kz)\right]\,, \tag{10}$$

$$\frac{d}{dt}(\gamma\dot{y}) = \frac{eE_0}{m}\left(1 + \tau\frac{d}{dt}\gamma\right)\left[\left(1 - \frac{\dot{z}}{c}\right)\sin(\omega t - kz)\right]\,, \tag{11}$$

$$\frac{d}{dt}(\gamma\dot{z}) = \frac{eE_0}{mc}\left(1 + \tau\frac{d}{dt}\gamma\right)\left[\dot{x}\cos(\omega t - kz) + \dot{y}\sin(\omega t - kz)\right]\,, \tag{12}$$

$$\frac{d}{dt}(\gamma c) = \frac{eE_0}{mc}\left(1 + \tau\frac{d}{dt}\gamma\right)\left[\dot{x}\cos(\omega t - kz) + \dot{y}\sin(\omega t - kz)\right]\,. \tag{13}$$

As one can see that right hand side of last two equations are identical, which means the left hand side of them should be equal each other. From this fact one can get

$$\dot{z} = c\left(1 - \frac{1}{\gamma}\right)\,, \qquad \rightarrow \qquad \frac{1}{\gamma} = 1 - \frac{\dot{z}}{c}\,. \tag{14}$$

Now integrating equations (10), (11), taking into account (14) and after simple algebraic manipulations one can have

$$\dot{x} = \frac{eE_0}{m\omega}\left(1 - \frac{\dot{z}}{c}\right)\left[\sin(\omega t - kz) + \omega\tau\cos(\omega t - kz) - \omega\tau\right]\,, \tag{15}$$

$$\dot{y} = \frac{eE_0}{m\omega}\left(1 - \frac{\dot{z}}{c}\right)\left[1 - \cos(\omega t - kz) + \omega\tau\sin(\omega t - kz)\right]\,, \tag{16}$$

and

$$x = \frac{eE_0}{m\omega^2}[1 - \cos(\omega t - kz)] - \frac{eE_0\tau}{m\omega}\left[\omega t - kz - \sin(\omega t - kz)\right]\,, \tag{17}$$

$$y = \frac{eE_0}{m\omega^2}\left[\omega t - kz - \sin(\omega t - kz)\right] + \frac{eE_0\tau}{m\omega}[1 - \cos(\omega t - kz)]\,. \tag{18}$$

Notice that here we have used the following initial conditions:

$$t = 0; \qquad x = y = z = 0; \qquad \dot{x} = \dot{y} = \dot{z} = 0\,. \tag{19}$$

The next step what we have to do is to find a solution for z. Now substituting expressions (15) and (16) into (12), one can have

$$\frac{d}{dt}(\gamma\dot{z}) = \frac{e^2E_0^2}{m^2\omega c}\left(1 + \tau\frac{d}{dt}\gamma\right)\left\{\left(1 - \frac{\dot{z}}{c}\right)\left[\sin(\omega t - kz) + \omega\tau - \omega\tau\cos(\omega t - kz)\right]\right\}\,, \tag{20}$$

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and integrating above equation one gets

$$\dot{z} = \frac{e^2 E_0^2}{m^2 \omega^2 c}\left(1 - \frac{\dot{z}}{c}\right)\{1 - \cos(\omega t - kz) + \omega\tau\,(\omega t - kz)\}\ . \tag{21}$$

Finally, the exact analytical expressions for the positions x, y, and z take the form:

$$x = r_0[1 - \cos(\omega t - kz)] - r_0\omega\tau\,[\omega t - kz - \sin(\omega t - kz)]\ , \tag{22}$$
$$y = r_0\,[\omega t - kz - \sin(\omega t - kz)] + r_0\omega\tau[1 - \cos(\omega t - kz)]\,, \tag{23}$$
$$z = \frac{1}{c}\omega r_0^2\left[\omega t - kz - \sin(\omega t - kz) + \frac{1}{2}\omega\tau\,(\omega t - kz)^2\right]\ , \tag{24}$$

where $r_0 = eE_0/(m\omega^2)$ is a characteristic amplitude of the oscillation. Consequently, the kinetic energy of particle in the external electromagnetic wave in the presence of radiation reaction force is

$$T = mc^2(\gamma - 1) = mc\gamma\dot{z} = \frac{e^2 E_0^2}{m\omega^2}\,[1 - \cos(\omega t - kz) + \omega\tau(\omega t - kz)]\ , \tag{25}$$

while the Lorentz factor is

$$\gamma = 1 + \left(\frac{eE_0}{m\omega c}\right)^2\{1 - \cos(\omega t - kz) + \omega\tau\,(\omega t - kz)\}\ . \tag{26}$$

Notice that in absence of the radiation reaction force, i.e. $\tau = 0$ solutions (22)-(24) read []

$$x = r_0[1 - \cos(\omega t - kz)]\,, \tag{27}$$
$$y = r_0\,[\omega t - kz - \sin(\omega t - kz)]\ , \tag{28}$$
$$z = \frac{1}{c}\omega r_0^2\,[\omega t - kz - \sin(\omega t - kz)]\ , \tag{29}$$
$$T = \frac{e^2 E_0^2}{m\omega^2}\,[1 - \cos(\omega t - kz)]\ , \tag{30}$$
$$\gamma = 1 + \left(\frac{eE_0}{m\omega c}\right)^2\{1 - \cos(\omega t - kz)\}\ . \tag{31}$$

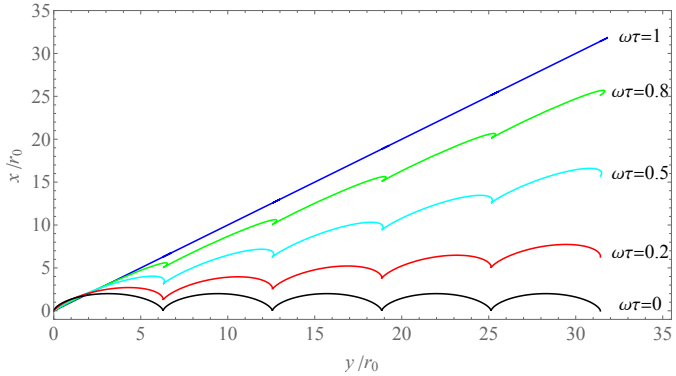


FIG. 1. The trajectory of charged particle in the external electromagnetic wave in the (x, y) plane for the different values of the damping parameter.

Figure 1 represents the trajectory of charged particle in (x, y) plane for the different values of the damping parameter. It is know that in absence of the radiation reaction force charged particle is governed by Lorentz equations and in this case charged particles trace periodic trajectory along y direction with the characteristic amplitude r_0 which shown in Fig. 1. However in the presence of the radiation reaction force the amplitude of the oscillation of particles decrease and they star to move also along x direction. Another interesting feature of charged particle motion in the presence of the radiation reaction force is that they move along the straight line in the very strong external electromagnetic wave represented in Fig. 1. Similarly, charged particle motion

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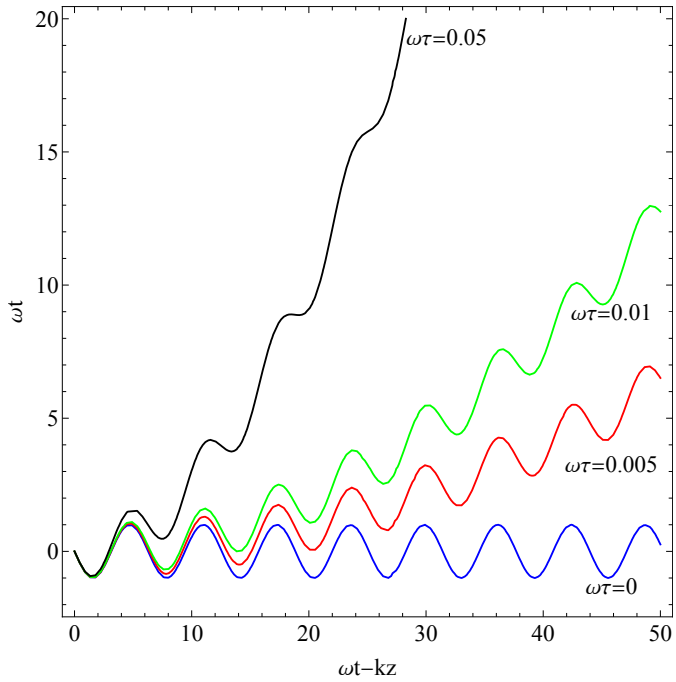


FIG. 2. The time dependence of the phase for the different values of the damping parameter.

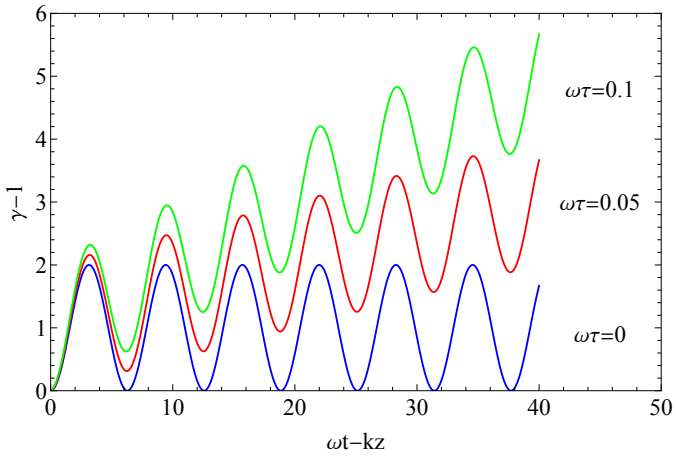


FIG. 3. The dependence of the Lorentz factor from the phase for the different values of the damping parameter.

along z direction, which is the same direction as the electromagnetic wave, is shown in Figure 2. As one see from figure that charged particles does not follow the wave direction in the presence of the radiation reaction force.

We also produce the dependence of the Lorentz factor from the phase of the electromagnetic wave in the presence of the radiation reaction force which is illustrated in Figure 3. As one see from figure that the Lorentz factor periodically varies as the the phase of the external electromagnetic wave. Due to the damping term in equation (1), charged particle can accelerate and its Lorentz factor increases with increases of the phase $\omega t - kz$.

The critical remarks are necessary concerning the application of the results of the problem. From the theoretical point of view the mechanism of the producing the high-energy cosmic ray and its physical origin has long been debated. It is widely believed that such high energy cosmic rays is generated due to acceleration of charged particle in the external electromagnetic field. However, in the present paper we have neglected the effects arise to gravitational field of the particle by the compact objects. Therefore, this method can be used for only rough estimations of the physical parameters in the processes of charged particle acceleration near the compact objects.

It is presented the detailed calculations charged particle acceleration in the presence of the circularly polarized electromagnetic

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wave taking into account radiation reaction force and ignoring the high order velocity terms. It has been explicitly shown how damping term affects on the trajectory of charged particles in the external electromagnetic field in the rectangular coordinates. We plan to consider charged particle acceleration by the electromagnetic wave produced by the magnetized neutron star in near future.

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