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Test particles motion in generalized Einstein-Maxwell-scalar theory

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In this paper, we explore the test particle motion around black hole in Einstein-Maxwell-scalar (EMS) theory. We analyze energy and angular momentum of test particles corresponding to circular orbits around the black holes together with their innermost stable circular orbits (ISCO) around the black hole. We obtain that the ISCO radius of the neutral particle decreases with the increase of black hole charge. Finally, we study the frequency of Keplerian orbits of the test particles around charged black holes in the EMS theory.

I. INTRODUCTION

The standard theory of gravitational field known as general relativity (GR) was introduced by Albert Einstein in 1915. GR as gravitational field theory has been successfully tested in both weak and strong field regimes [1–4]. However, GR meets several fundamental problems which cannot be resolved within the basic principles of the theory. One of the above-mentioned problems is the existence of the singularity at the origin of most general relativistic solutions. On the other hand, the current resolutions of experiments and observations leave open the window for the development of alternative/modified theories of gravity, which can be further used to resolve the fundamental open issues of gravitational interaction. One of the possible ways of extension of GR is to consider the low energy limit of st ring theory, where one may introduce the dilaton scalar field. This field will be included in the action as an additional supplement term to the Einstein–Hilbert action and has the form of the axion, gauge fields, and another non-trivial coupling of dilaton to fields. Particularly, some aspects of such combination like the causal structures and thermodynamic properties of black hole (BH) solution with dilaton have been investigated in Refs. [5–8]. Further, one may extend the black hole solution using the cosmological constant. The models with the presence of negative cosmological constants are prominent candidates for gravity theories defined in higher dimensions. Detailed analysis of these models and corresponding solutions can be found in Refs. [9–11]. One may also consider the heterotic string theory, where the scalar dilaton field is coupled to the electromagnetic field tensor [6]. Here, we plan to study the properties of the black hole solution within the Einstein–Maxwell-scalar (EMS) field theory using the dynamics of test particles. The tests of the field theories based on the observations of compact objects allow us to get the constraints on the field parameters. Particularly, the modified and alternative theories of gravity and corresponding solutions describing the compact objects have been successfully tested using X-ray data from astrophysical objects by the Authors of Refs. [12–14]. One may also develop the tests of the general relativity and other models of gravitational interaction based on the test particle motion.
One of the interesting features of the dynamics of test particles appears with the analysis of circular/bound orbits around compact objects.

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II. BLACK HOLES IN EINSTEIN-MAXWELL-SCALAR THEORY

The exact analytical solutions for space-time around the static black hole in general form have been found in Ref. [15] in the following form,

$$ds^2 = -U(r)dt^2 + \frac{1}{U(r)}dr^2 + f^2(r)(d\theta^2 + \sin\theta^2 d\phi^2) \tag{1}$$

where $U(r)$ and $f(r)$ are radial functions. Assume the time component of the vector and the dilaton fields depend on the radial coordinate only, i.e., $A_t = A_t(r)$ and $\phi = \phi(r)$.

$$U(r) = (a - \frac{b}{r})(1 - \frac{a}{r})^{1-\gamma} + \frac{\beta Q^2}{f^2} \tag{2}$$

$$f(r) = r(1 - \frac{a}{r})^{\frac{\gamma}{2}} \tag{3}$$

with

$$M = \frac{1}{2}[b + (1-\gamma)a], \qquad Q^2 = (1 - \frac{\gamma}{2})ab \tag{4}$$

III. GEODESIC MOTION

In this section, we study test particle motion around the black hole in the Einstein-Maxwell-scalar theory. Since the space-time metric (1) is independent of coordinates t and ϕ, then corresponding momentum to these coordinates, p_t and p_ϕ should be conserved, which are related to the energy and the angular momentum of a test particle at infinity. It is well known that the four-velocity of test particle, i.e., $u^\alpha = \dot{x^\alpha} = dx^\alpha/d\lambda$, where λ is an affine parameter, is satisfied to the following normalization condition: $g_{\alpha\beta}u^{\alpha u^\beta} = -1$ which allows to write equation of motion in background geometry (1) in the form

$$\dot{r}^2 + U(r)f(r)^2\dot{\theta}^2 = \varepsilon^2 - V_{eff}(r,\theta) \tag{5}$$

where the effective potential $V_{eff}(r,\theta)$ is defined as

$$V_{\text{eff}}(r,\theta) = U(r)\left(1 + \frac{\mathcal{L}^2}{f(r)^2\sin^2\theta}\right) \tag{6}$$

here ε and $\mathcal{L}$ are, respectively, the specific energy and the specific angular momentum of a test particle at infinity. Notice that these quantities satisfy the following relations:

$$\mathcal{E} = U(r)u^t, \qquad \mathcal{L} = f(r)^2\sin^2\theta u^\phi \tag{7}$$

The graph of effective potential of the test particles around EMS BH is described in Fig.1. Moreover, the radial dependence of specific energy and angular momentum of the test particles around EMS BH is represented in Fig.2.

Now we focus on the analysis of test particle motion by investigating the effective potential. The standard analysis shows that particle motion is limited by the energetic boundary conditions given by $\dot{r} = 0$ and $\dot{\theta} = 0$ which leads to

$$V_{eff}(r,\theta) = 0 \tag{8}$$

On the other hand, the stationary points of the effective potential (6)can be found from the following conditions

$$\partial_r V_{eff}(r,\theta) = 0, \qquad \partial_\theta V_{eff}(r,\theta) = 0 \tag{9}$$

which allow finding extreme (r_0 , θ_0) of the function.

The radius of innermost stable circular orbits (ISCO) of test particles can be defined using an additional condition $V''_{eff} \geq 0$. We have not wrote exact form of ISCO radius, but we have been plotting its graphs in Fig. 3

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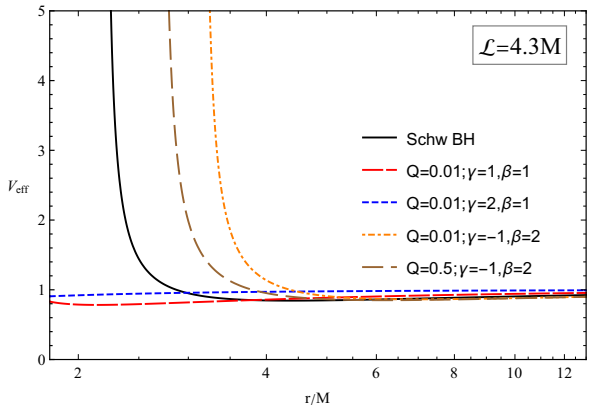


FIG. 1. Profiles of the effective potential for the radial motion of test particles around charged BHs in EMS gravity for various values of the EMS parameters Q, β and γ .

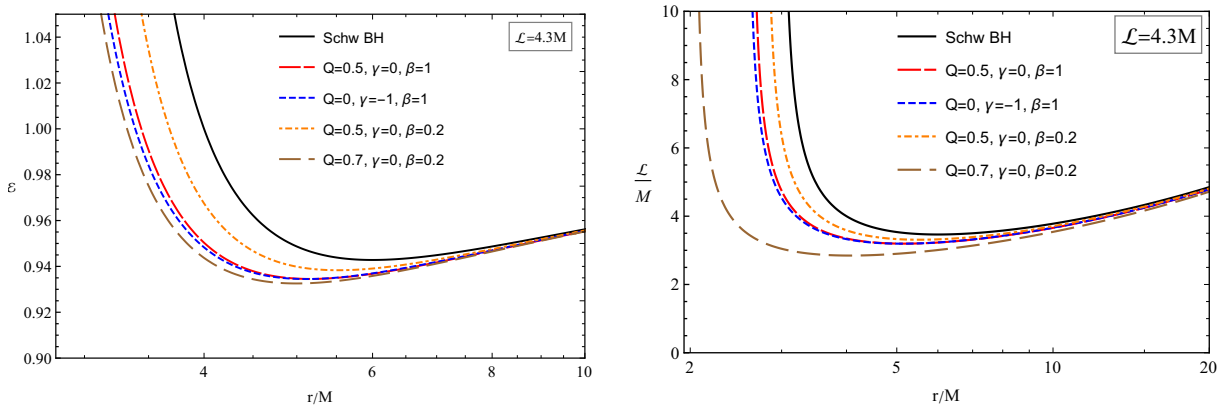


FIG. 2. The radial dependence of specific energy (left panel) and angular momentum (right panel) of test particles around EMS BHs for different values of the parameters Q, β and γ.

IV. FUNDAMENTAL FREQUENCIES

In this section, we will show in detail the derivation of expression for the fundamental frequencies governed by test particle orbiting around the black hole in the Einstein-Maxwell-scalar theory which is one of the sim- ple models to explain Quasi-Periodic oscillations (QPO) observable around compact astrophysical objects. The angular velocity measured by a distant observer can be found as

$$\Omega = \sqrt{-\frac{\partial_r g_{tt}}{\partial_r g_{\phi\phi}}} = \sqrt{\frac{U'(r)}{(f^2(r))'}} \tag{10}$$

The radial profiles of the Keplerian frequencies of test particles around charges EMS BHs are presented in Fig.4 for different values Q, β and γ.

However, there might exist other types of oscillations characterized by the radial and vertical frequencies. These radial and vertical fundamental frequencies can be calculated by considering small perturbation around the circular orbit, respectively, along with the radial $r \to r_0 + \delta r$ and azimuthal $\theta \to \theta_0 + \delta\theta$

V. CONCLUSION

In this work, we have studied test particle motion around black hole in Einstein-Maxwell-scalar (EMS) theory. Moreover, we have discussed effective potential, and we have found fundamental frequencies. From the performed research we summarize our main results as:

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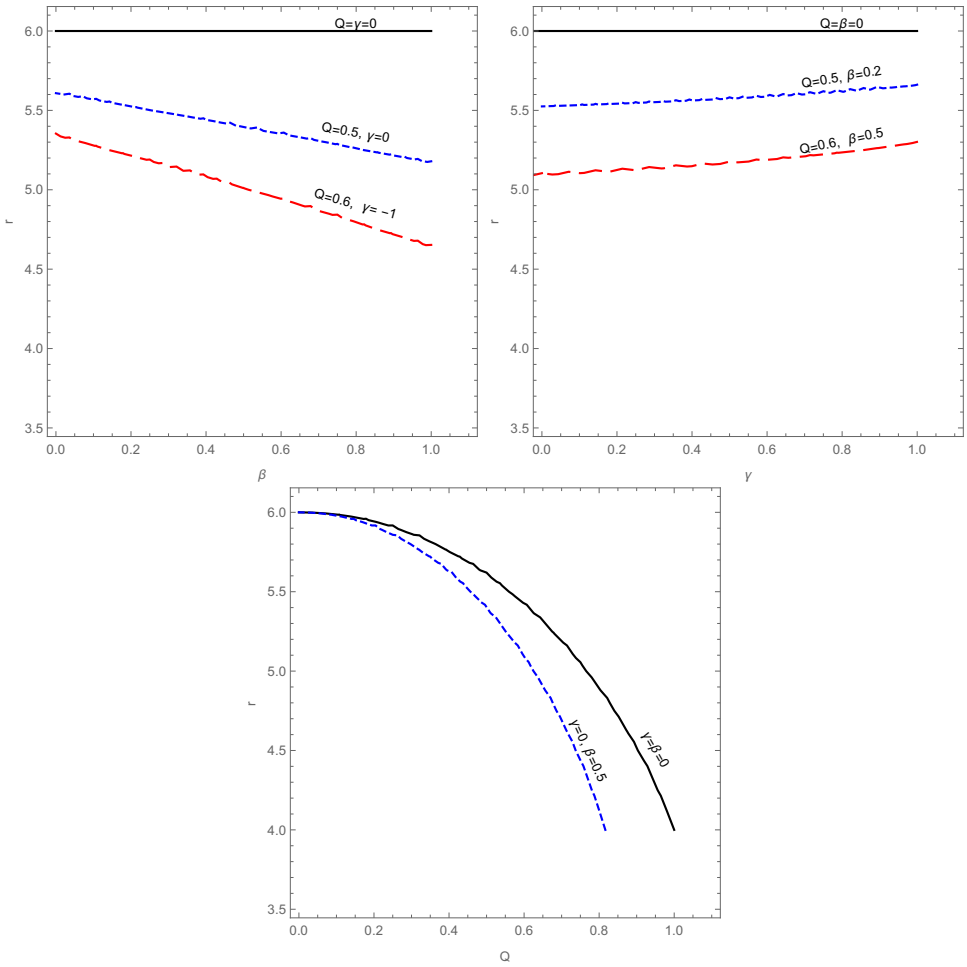


FIG. 3. Dependence of ISCO radius on EMS BH parameters.

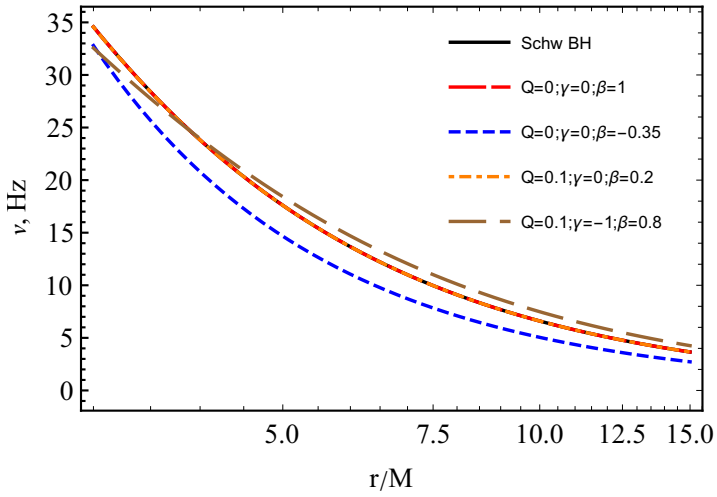


FIG. 4. Keplerian frequency of test particles around charges BHs in EMS gravity as a function of the radial coordinate for various values of the EMS parameters Q, β and γ.

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- We have investigated the dynamics of test particles in the spacetime of charged BHs in EMS theory. Analytic expressions for the specific angular momentum and energy of test particles at circular orbits are found, and the effects of the parameters Q, β and γ on them are discussed. It has been shown that an increase of the BH charge causes decreasing the minimum of both energy and angular momentum, however, the presence of γ and β parameters increases.
- We have also analyzed EMS field effects on ISCO radius of the test particles around EMS BH in Fig.3.
- Finally, we have investigated fundamental frequencies. and we have plotted the graph of the Keplerian frequencies in Fig.4.

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