Journal of Fundamental and Applied Research **Vol. 2**, Issue 4 (2022) 20220026

Received: 20 November 2022 / Accepted: 11 December 2022
Published online: 25 December 2022
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Curvature of Specetime Around KNNK BH in the Rastall Gravity

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In the work we investigate the spacetime properties of so called Kerr-Newman-NUT-Kiselev black hole (KNNK BH) in the Rastall theory of gravity. We introduce the spacetime metric that describes KNNK BH and calculate the Kretschmann scalar that defines the curvature properties around gravitating objects. We show the effect of spacetime parameters involved in the given metric on the Kretschmann scalar.

I. INTRODUCTION

The spacetime around Kerr-Newman-NUT-Kiselev black hole is governed by the following metric [1]

$$ds^2 = -\frac{\Delta}{\Sigma}[dt - \{a\sin^2\theta + 2l(1-\cos\theta)\}d\phi]^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}[adt - \{r^2 + (a+l)^2\}d\phi]^2\,, \tag{1}$$

where the following notations have been used

$$\Delta = r^2 - 2Mr + a^2 + e^2 + g^2 - l^2 - \alpha r^v, \tag{2}$$

$$v = \frac{1-3\omega}{1-3\kappa\lambda(1+\omega)}, \tag{3}$$

$$\Sigma = r^2 + (l + a\cos\theta)^2\ . \tag{4}$$

It is worth noting here that in our previous published works devoted to the investigation of the properties of KNNK BH in the Rastall gravity (RG) we have studied particle motion and QPOs [2] and optical properties of KNNK BH in the RG [3].

In the given spacetime metric M is a mass parameter, a is spin of a black hole, e is electric charge, g is magnetic charge, l is NUT parameter or gravitomagnetic charge, α is the quintessential intensity, ω is the parameter of equation of state of the quintessence and $\kappa\lambda$ is Rastall parameter. One can check that in the absence of the quintessential intensity, α and NUT parameter, l the given spacetime recovers Kerr-Newman spacetime with electromagnetic charge. Determinant of the metric is

$$g = -\frac{1}{4}\sin^2\theta\left(a^2\cos 2\theta + a^2 + 4al\cos\theta + 2l^2 + 2r^2\right)^2\ . \tag{5}$$

The curvature of spacetime is usually described by Riemann tensor. But we know that tensor quantities are coordinate dependent and thus it is better to use a scalar formed from the Riemann tensor. Such scalar is called the Kretschmann scalar which can be written as the square of the Riemann tensor, $\mathcal{K} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$. The explicit form of such scalar for $\theta = \pi/2$ takes the following form

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$$\mathcal{K} = \frac{1}{r^4\left(l^2+r^2\right)^6}\Bigg[l^8\left(\alpha^2(v-1)^2v^2r^{2v}-8\alpha(v-1)vr^{v+2}+48r^4\right)-4l^6r^2\left(12M^2r^2-6Mr\left(24r^2-\alpha v(v+1)r^v\right)\right.$$
$$+q^2\left(24r^2-\alpha(v-1)vr^v\right)+\alpha^2\left(-v^4\right)r^{2v}+4\alpha^2v^3r^{2v}+\alpha^2v^2r^{2v}-2\alpha v^2r^{v+2}-\alpha^2vr^{2v}-46\alpha vr^{v+2}-24\alpha r^{v+2}$$
$$\left.+180r^4\right)+2l^4r^4\left(360M^2r^2-2q^2\left(120Mr+\alpha\left(v^2+31v+28\right)r^v-240r^2\right)-20Mr\left(\alpha\left(v^2-7v-12\right)r^v+48r^2\right)\right.$$
$$+28q^4+3\alpha^2v^4r^{2v}-18\alpha^2v^3r^{2v}+15\alpha^2v^2r^{2v}+20\alpha v^2r^{v+2}+62\alpha^2vr^{2v}+28\alpha^2r^{2v}-20\alpha vr^{v+2}-480\alpha r^{v+2}+360r^4\Big)$$
$$-4l^2r^6\left(180M^2r^2+q^2\left(-240Mr+\alpha\left(5v^2+11v-136\right)r^v+120r^2\right)+2\alpha M\left(v^2-31v+120\right)r^{v+1}-144Mr^3\right.$$
$$\left.+68q^4+\alpha^2\left(-v^4\right)r^{2v}+8\alpha^2v^3r^{2v}-19\alpha^2v^2r^{2v}-6\alpha v^2r^{v+2}-11\alpha^2vr^{2v}+68\alpha^2r^{2v}+54\alpha vr^{v+2}-120\alpha r^{v+2}+12r^4\right)$$
$$+r^8\left(48M^2r^2-4q^2\left(24Mr+\alpha\left(3v^2-19v+28\right)r^v\right)+8\alpha M\left(v^2-7v+12\right)r^{v+1}+56q^4\right.$$
$$\left.\left.+\alpha^2\left(v^4-10v^3+41v^2-76v+56\right)r^{2v}\right)\right]. \tag{6}$$

One can see that explicit form of the Kretschmann scalar is quite complicated even in the case of $\theta = \pi/2$ and implicit form of v. It is also noticeable that the Kretschmann scalar does not depend on the spin parameter when calculated in the equatorial plane which is common with the Kerr spacetime. It can be also easily checked that the Kretschmann scalar tends to zero when $r- > \infty$ in the equatorial plane. In the above equation electromagnetic charge q is defined as $q^2 = e^2 + g^2$.

One can plot the radial dependence of the Kretschmann scalar for different values of the spacetime parameters as presented in Fig. 1 in the close environment of the origin ($r = 0$) of a black hole. From the top-left panel it can be seen that the curvature scalar tends to zero from the negative values part which indicates that the resulting force becomes repulsive in the region $r \gtrsim 0.25M$ for the given values of the spacetime parameters. The resulting force then becomes attractive in the range $0.08M \lesssim r \lesssim 0.25M$ since the curvature scalar is positive in such range. In the range $r \lesssim 0.08M$ the curvature scalar becomes negative again and tends to $-\infty$ suggesting that the there is a physical singularity in the case $r \to 0$. It can be also seen that increase of the intensity of quintessence α the lines slightly contracts horizontally and enlarges vertically. Similar tendency of lines can be observed in other panels as well i.e. in all cases the lines start from $-\infty$, go up exponentially, reach maximum point in positive values, then start going down up to some minimum in negative values and finally tend to zero from the negative values with the increase of the radial coordinate. In the top-right panel it s shown that increase of the electromagnetic charge shifts the lines towards negative values for smaller values of the radial coordinate ($0 < r \lesssim 0.3M$) while for relatively far distances from the center ($0.3M \lesssim r \lesssim 0.5M$) shifts the lines towards positive values. From the middle-left panel it can be observed that effect of the NUT parameter l is much stronger as it drastically changes the behavior of the lines. From the middle-right panel one can clearly see that the effect of the parameter of equation of state of the quintessence ω also changes the curvature profile considerably and it can be also seen that the difference between the cases $\omega = -1/10$ and $\omega = -1/2$ is considerably large compared to the difference between $\omega = -1/2$ and $\omega = -2/3$. From the last panel it can be seen that increase of the Rastall parameter shifts the positive part of the lines upward while the negative part around the singularity is shifted downward starting from some specific values of the radial coordinate.

Conclusion

In the current work curvature properties of KNNK BH in the Rastall gravity has been studied. It has been shown the explicit form of so called Kretschmann scalar that defines the curvature and independent of the choice of coordinates. It has been checked that the Kretschmann scalar tends to zero when the radial coordinate tends to infinity. Radial dependence of this scalar indicated that effects of parameters α is considerably weaker compared to the rest spacetime parameters. It has been demonstrated that effect of the NUT parameter is the strongest one but only in the close

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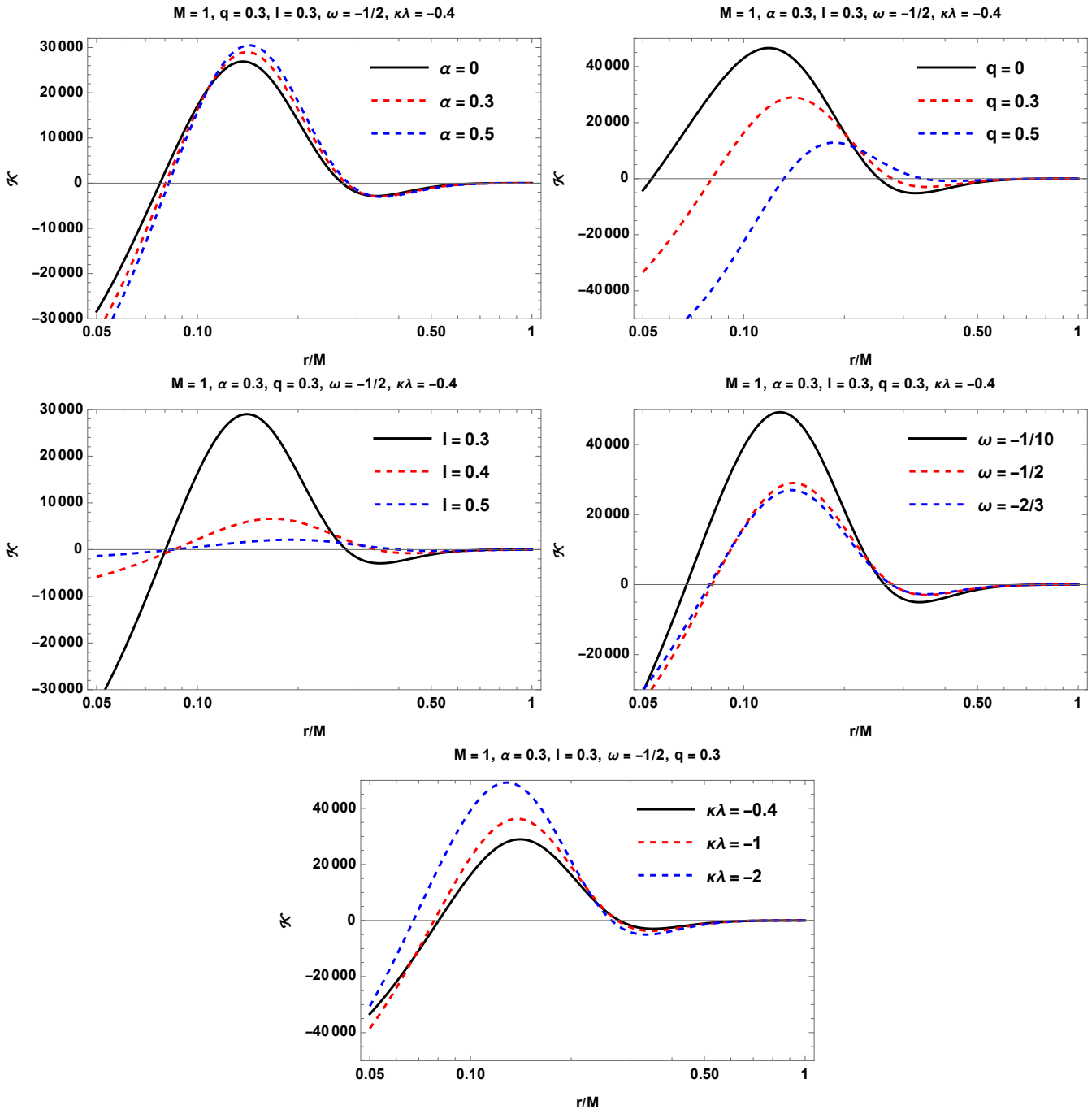


FIG. 1: Change of the Kretschmann scalar with the change of radial coordinate for different values of the spacetime parameters.

environment of the singularity that appears at $r \to 0$. The effect of the electromagnetic charge is also strong in the mentioned ranges of the radial coordinate.

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