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Dynamics of electrically charged and magnetized particles around regular Ayón-Beato-Garcea black holes in the presence of external magnetic fields

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In this work we study electrically charged and magnetized particles (later only particles) around Ayón-Beato-Garcia black hole and external magnetic field. Ayón-Beato-Garcia black hole is a regular black hole, in our case, it has mass (M) and electric charge (q). Interestingly, we test these particles' motion with and without an external magnetic field, and the effective potential of these particles is affected by the external magnetic field and charge of these particles. We found that for the particles which travel in circular orbit as Ayón-Beato-Garcia BH's charge increases, it causes to decrease angular momentum of these particles. We also study QPOs of these particles. QPOs are affected by the external magnetic fields, an increase in the magnetic field affects to decrease in the radial distance of fundamental frequencies.

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I. INTRODUCTION

The celebrated singularity theorems proved by Penrose and Hawking [1] claim that under some circumstances, the existence of singularities is inevitable in General Relativity. This is in accordance with the observation that the first known exact black hole solutions in General Relativity have a singularity inside the event horizon. However, it is widely believed that singularities are non-physical objects which are created by classical theories of gravity, and they do not exist in nature. In fact, the quantum arguments given by Sakharov [2] and Gliner [3] suggest that the space-time singularities could be avoided for matter sources with a de Sitter core at the center of space-time. Based on this idea, Bardeen proposed the first static spherically symmetric regular black hole solution [4]. Other regular black hole models are also proposed later [5–14]. It is easily shown that all these regular black hole models violate the strong energy condition[21] and hence can break the singularity theorems.

It was established by Ayón-Beato and García [15–17] that the regular black hole models can be interpreted as the gravitational field of a nonlinear electric or magnetic monopole. Thus, the physical source of the regular black holes could be a nonlinear electromagnetic field. This is also ensured by other authors in the literature [18]. Recently, it was shown in [19] that some regular black hole solutions can be constructed in $f(T)$ gravity coupled to a nonlinear electrodynamics.

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In this work, we plan to extend these results to the charged magnetized particle motion and acceleration process around a static regular black hole immersed in an external uniform magnetic field (without ergosphere) and study the influence of electric charge of the black hole on these processes.

Throughout this work we use signature $(-,+,+,+)$ for the space-time and geometrized unit system $G = c = 1$ (However, for an astrophysical application we have written the speed of light explicitly in our expressions). Latin indices run from 1 to, 3 and Greek ones from 0 to 3.

II. CHARGED PARTICLE MOTION

The line element of regular Ayón-Beato-Garcia black holes is given in. as

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + d\Omega^2 \tag{1}$$

where

$$f(r) = 1 - \frac{2Mr^2}{(r^2+q^2)^{3/2}} + \frac{q^2r^2}{(q^2+r^2)^2} \tag{2}$$

with the associated electric field E is given by

$$E_r(r) = qr^4\Big[\frac{r^2-5q^2}{(r^2+q^2)^4} + \frac{15}{2}\frac{M}{(r^2+q^2)^{7/2}}\Big]$$

One can find the four potentials of the electromagnetic field around the black hole generated by nonlinear electrodynamics charge of the black holes by integrating the electric field strength as $A_t = \int E_r dr$ and we get

$$A_t = \frac{qr^5}{2}\left[\frac{3M}{q^2\left(q^2+r^2\right)^{5/2}} - \frac{2}{\left(q^2+r^2\right)^3}\right] + C$$

here C is integral constant. One can find the constant using limit when $q \to 0$ scalar electric potential also should be zero.

$$\lim_{q\to 0}\frac{qr^5}{2}\left(\frac{3M}{q^2\left(q^2+r^2\right)^{5/2}} - \frac{2}{\left(q^2+r^2\right)^3}\right) + C = \frac{3M}{2q} + C$$

then $C = -\frac{3M}{2q}$

A. External magnetic field

In this subsection we find four-potential of electromagnetic field around the Ayýon Beato Garcia black hole immersed in an external asymptotically uniform magnetic field using Wald's method [20],

$$A_\phi = \frac{1}{2}Br^2\sin\theta, \tag{3}$$

$$\begin{aligned} A_t &= \frac{1}{2}\left[\frac{3Mr^5}{q\left(q^2+r^2\right)^{\frac{5}{2}}} - \frac{2qr^5}{\left(q^2+r^2\right)^3}\right] - \frac{3M}{2q} \\ &= \Big[\frac{3M}{2q}\frac{\sqrt{r^2+q^2}}{r} - \frac{q}{r}\Big]\left(\frac{r^2}{r^2+q^2}\right)^3 - \frac{3M}{2q} \end{aligned} \tag{4}$$

and immediately we can have expressions for the non-zero components of the electromagnetic tensor are

$$F_{r\phi} = Br\sin^2\theta \tag{5}$$

$$F_{rt} = \frac{qr^4}{(r^2+q^2)^3}\Big[\frac{r^2-5q^2}{r^2+q^2} + \frac{15}{2}\frac{M}{(r^2+q^2)^{\frac{1}{2}}}\Big] \tag{6}$$

$$F_{\theta\phi} = Br^2\sin\theta\cos\theta \tag{7}$$

nonzero components of magnetic field

$$B^{\hat{r}} = B\cos\theta, \qquad B^{\hat{\theta}} = \sqrt{f(r)}B\sin\theta \tag{8}$$

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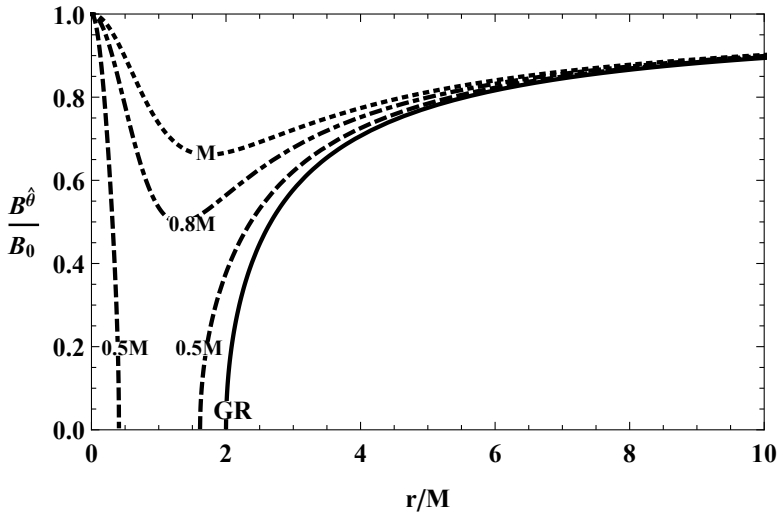


FIG. 1: Radial profile of $B^{\hat{\theta}}$

B. Equations of motion

Here we will study the motion of a charged particle around a Regular Black Hole using the Hamilton-Jacobi equation

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^{\mu}}+eA_{\mu}\right)\left(\frac{\partial S}{\partial x^{\nu}}+eA_{\nu}\right)=-m^{2} \tag{9}$$

Since, t and ϕ are the killing variables, the cation can be written in the following form

$$S=-\mathcal{E}t+l\phi+S_{r}(r)+S_{\theta}(\theta) \tag{10}$$

One can easily find Effective potential for the particle by putting Eq.10 into Eq.9 in the following simple form

$$V_{\text{eff}}=Q(r)+\sqrt{f(r)\left[1+\left(\frac{l}{r}+\omega_{B}r\right)^{2}\right]} \tag{11}$$

here denotes $Q(r)=\frac{e}{m}A_{t}$ and $\omega_{B}=\frac{eB}{2m}$.

The particle motion is limited by the energetic boundaries given by

$$\mathcal{E}=V_{\text{eff}}(r) \tag{12}$$

the stationary points of the effective potential $V_{\text{eff}}(r)$ function, where maximum or minimum can exist, are given by the equations

$$\partial_{r}V_{\text{eff}}=\frac{f'(r)\left(\left(\frac{l}{r}+r\omega_{B}\right)^{2}+1\right)+f(r)\left(2r\omega_{B}^{2}-\frac{2l^{2}}{r^{3}}\right)}{2\sqrt{f(r)\left(\left(\frac{l}{r}+r\omega_{B}\right)^{2}+1\right)}}-\frac{Q}{r^{2}} \tag{13}$$

FIG. 2 gives information about the effective potential of the specific charged particle $e/m=10^{6}$ for different values of the charge of the Ayón-Beato-Garcia BH. Generally speaking, as the charge of the BH increases, the value of the effective potential decreases, and there are three panels. The left one illustrates the effective potential of the specific charged particle without an external magnetic field while in the middle panel, the particle is in an external magnetic field ($B=10^{5}G$) and the external magnetic field affects the particles where far from ISCO points. In the last panel the external magnetic field is $B=2\cdot10^{5}G$.

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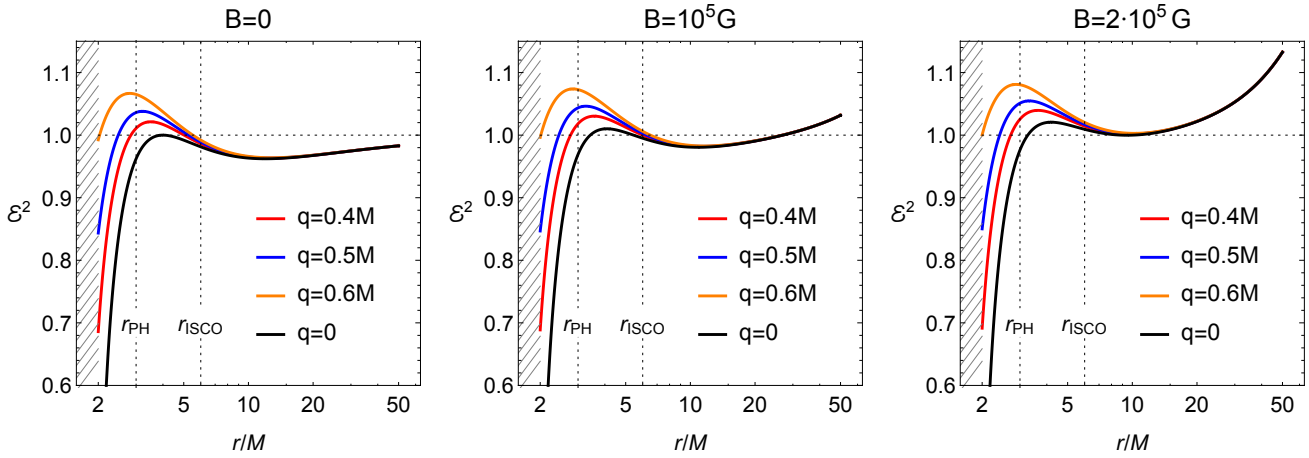


FIG. 2: Radial profiles of the effective potential in equatorial plane V_{eff} for the various values of black hole charge and external magnetic field, the value of specific charge of the particle $e/m = 10^{-6}$

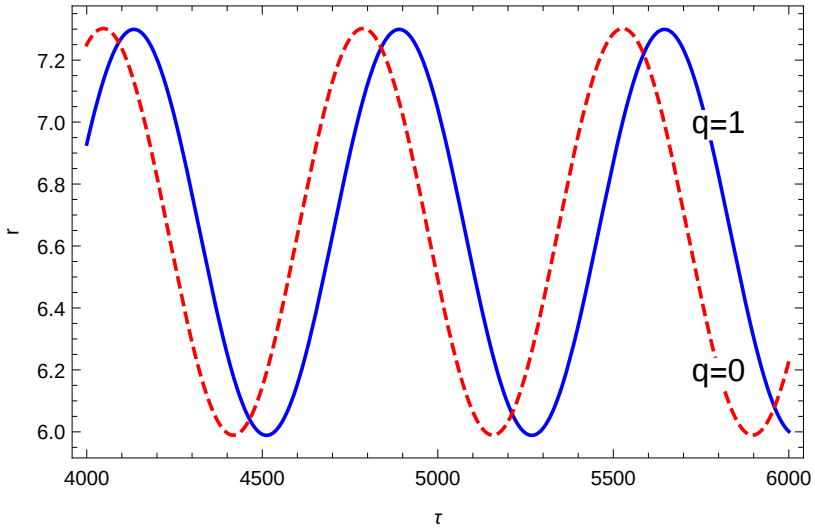


FIG. 3: Radial oscillations of the charged particle at the stable orbit, here also we have chosen $q/m = 10^{-6}$ for the charged particle and magnetic field is $B = 10^4 G$

C. Geodesic equation

Geodesic equation for the charged particle can write as

$$\ddot{t} + (\log f(r))' \dot{r}\dot{t} + \frac{Q}{r^2 f(r)} \dot{r} = 0 \tag{14}$$

$$\ddot{\phi} + \left(2\dot{\phi} + \omega_B\right) \frac{\dot{r}}{r} = 0 \tag{15}$$

$$\ddot{r} + \frac{1}{2}(\log f(r))' \dot{r}^2 + \frac{1}{2}(f^2(r))' \dot{t}^2 - r f(r) \dot{\phi}^2 = \\ = -\omega_B r f(r) \dot{\phi} + \frac{Q}{r^2} \dot{t} \tag{16}$$

here we use $\dot{x}$ is partial derivative by affine parameter, and $'$ is by radial coordinates

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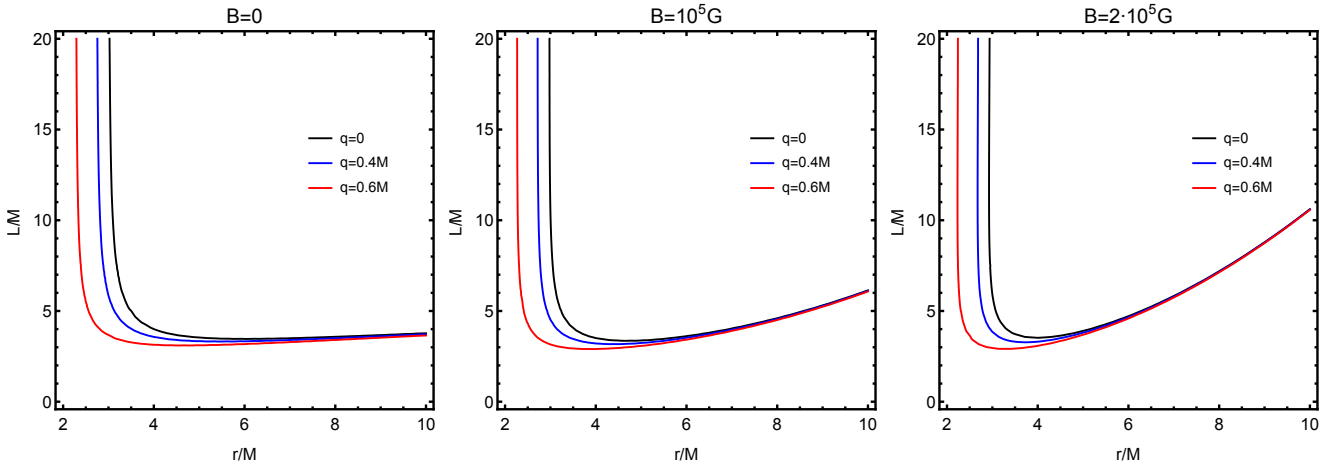


FIG. 4: The charged particle circular orbits at the equatorial plane with different values of black hole charge an external magnetic field for the value for the special charge of the particle $|e|/m = 10^{-6}$, Here thick black, red dashed and blue dot-dashed lines correspond to the cases $q = 0$, $q = 0.4M$ and $q = 0.6M$, here $x = r\cos\phi$ and $y = r\sin\phi$

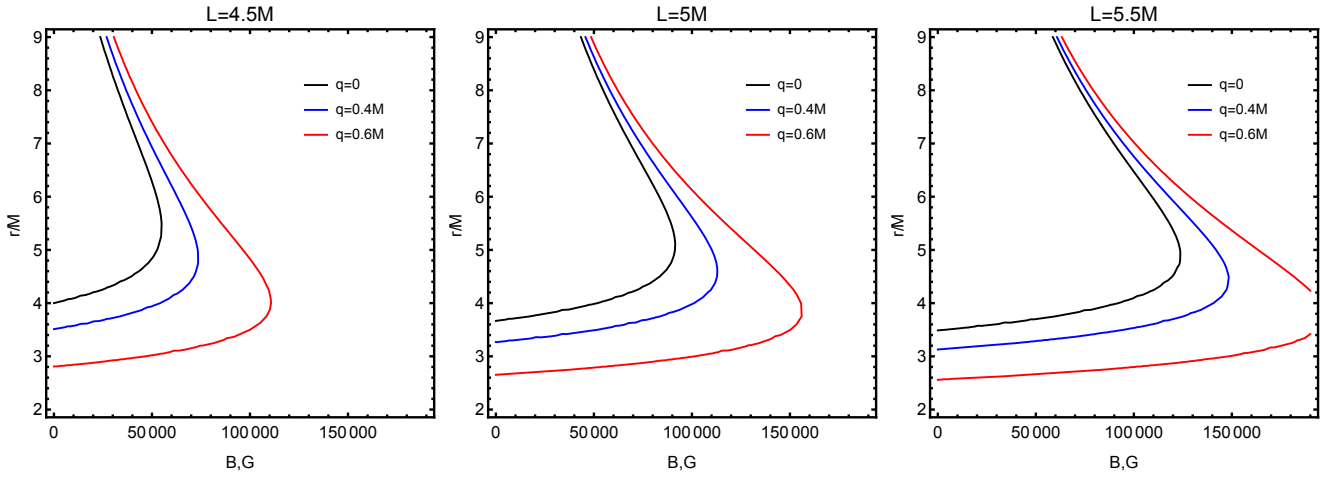


FIG. 5: The charged particle circular orbits at the equatorial plane with different values of black hole charge and external magnetic field for the value for the special charge of the particle $|e|/m = 10^{-6}$, Here thick black , red dashed and blue dot-dashed lines correspond to the cases $q = 0$, $q = 0.4M$ and $q = 0.6M$, here $x = r\cos\phi$ and $y = r\sin\phi$

D. Circular orbits

It is seen from the expression of effective potential that is not possible to find circular orbits analytically. Here we show numerically obtained results. To derive circular orbit and angular momentum at the circular orbit of the particle one should solve Eq. 12 and Eq. 13 equations at the same time. In FIG. 3, the trajectory of the radial oscillations of the charged particle is illustrated for the different values of the charge of the Ayón-Beato-Garcia BH. We can see from the plot that as the BH is charged it affects the initial phase of the oscillation, not the amplitude of the oscillation. Moreover, in FIG. 4 dependence of the angular momentum to the radial distance for the different values of the charge of the Ayón-Beato-Garcia BH is shown. FIG. 5 expresses the dependence of the circular orbit on the external magnetic field for the different values of BH charge, black line for $q = 0$, blue line for $q = 0.4M$ and red line for $q = 0.6M$ and in all case, angular momentum is constant and equals to $L = 4.5M$, $L = 5M$ and $L = 5.5M$ respectively. The last, the dependence of the circular orbit on the BH charge for the different values of the external magnetic field is expressed in FIG. 6.

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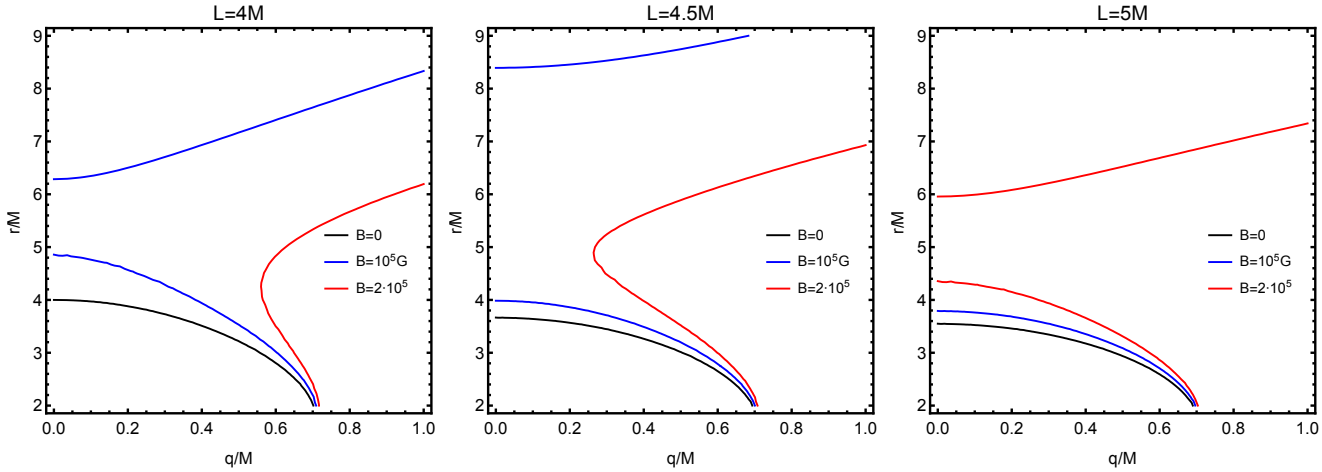


FIG. 6: The charged particle circular orbits at the equatorial plane with different values of black hole charge and external magnetic field for the value for the special charge of the particle $|e|/m = 10^{-6}$, Here thick black , red dashed and blue dot-dashed lines correspond to the cases $q = 0$, $q = 0.4M$ and $q = 0.6M$, here $x = r\cos\phi$ and $y = r\sin\phi$

III. QPO'S OF CHARGED PARTICLE AROUND REGULAR BLACK HOLE IMMERSED IN MAGNETIC FIELD

If a charged test particle is slightly displaced from the equilibrium position located in a minimum of the effective potential $V_{eff}(r,\theta)$ at r_0 and $\theta_0 = \pi/2$, corresponding to a stable circular orbit, the particle will start to oscillate around the minimum realizing thus epicyclic motion governed by linear harmonic oscillations. For harmonic oscillations around the minima of the effective potential V_{eff}, the evolution of the displacement coordinates $r = r_0+\delta r$, $\theta = \theta_0+\delta\theta$ is governed by the equations

$$\ddot{\delta r} + \omega_r^2 \delta r = 0 \qquad \ddot{\delta\theta} + \omega_\theta \delta\theta = 0 \tag{17}$$

locally measured angular frequencies of the harmonic oscillatory motion are given by [41]

$$\omega_r^2 = \frac{\partial V_{eff}}{\partial r^2} \tag{18}$$

$$\omega_\theta^2 = \frac{1}{r^2 f(r)} \frac{\partial V_{eff}}{\partial \theta^2} \tag{19}$$

$$\begin{aligned}\omega_r^2 = {} & \frac{2Q}{r^3} - \sqrt{\frac{1+\left(\frac{l}{r}-\frac{r\omega_B}{2}\right)^2}{4f(r)}} f''(r) \\ & + \frac{\left(f'(r)\left(1+\left(\frac{l}{r}-\frac{r\omega_B}{2}\right)^2\right)+2f(r)\left(\frac{r\omega_B^2}{4}-\frac{l^2}{r^3}\right)\right)^2}{4\left(f(r)\left[1+\left(\frac{l}{r}-\frac{r\omega_B}{2}\right)^2\right]\right)^{3/2}} \\ & - \frac{f'(r)\left(\omega_B^2 r-\frac{4l^2}{r^3}\right)+f(r)\left(\frac{6l^2}{r^4}+\frac{\omega_B^2}{2}\right)}{2\sqrt{f(r)\left(1+\left(\frac{l}{r}-\frac{r\omega_B}{2}\right)^2\right)}}\end{aligned} \tag{20}$$

$$\omega_\theta^2 = \left(\frac{l^2}{r^2} - \frac{\omega_B^2}{4}\right)\sqrt{\frac{r^2 f(r)}{1+\left(\frac{l}{r}-\frac{r\omega_B}{2}\right)^2}} \tag{21}$$

$$\omega_\phi = \frac{l}{r} - \frac{1}{2}\omega_B \tag{22}$$

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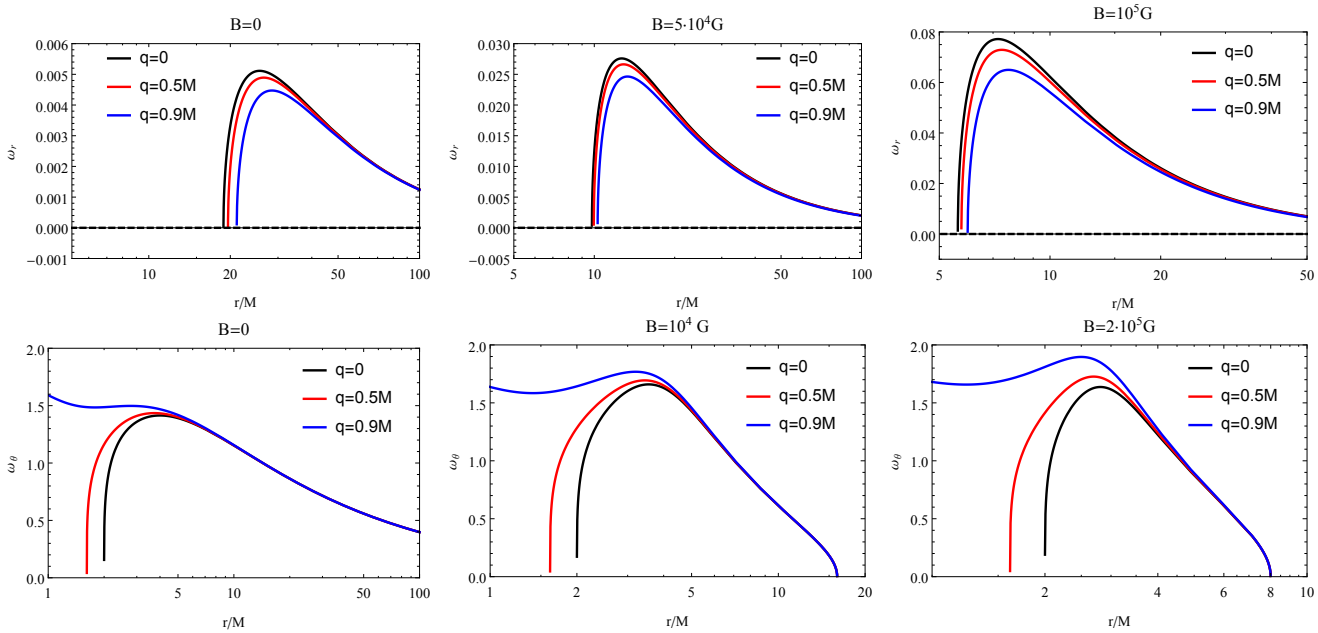


FIG. 7: Radial profiles of the locally measured fundamental frequencies

FIG. 7 shows frequencies of the radial (top panels) and the vertical (bottom panels) oscillations of charged test particles around charged Ayón-Beato-Garcia BH for various values of charge q BH and B external magnetic field for the fixed value of the particle charge as $e/m = 10^{-6}$. One can see from the figure that the frequency decreases in the presence of the Ayón-Beato-Garcia BH charge. Moreover, the external magnetic field causes to increase in frequencies and the left panel when $B = 0$, middle-upper, and lower are for $B = 5 \cdot 10^4 G$ and $B = 10^4 G$ respectively. And the right upper and lower are for the $B = 10^5$ and $B = 2 \cdot 10^4$ respectively.

IV. SUMMARY AND DISCUSSIONS

In this work, we have studied the effective potential of charged particles around charged Ayón-Beato-Garcia BH in presence of the external magnetic field. It is found that in increase BH charge causes to increase in the effective potential and it is obtained for $q = 0, q = 0.4M, q = 0.5M$, and $q = 0.6M$ values of the Ayón-Beato-Garcia BH charge.

Moreover, radial and vertical frequencies also are investigated in presence of the external magnetic field. One can see for the $B = 0, B = 5 \cdot 10^4$, and $B = 10^5$ values of the external magnetic field values, the frequencies shifts to a closer radial region to the Ayón-Beato-Garcia BH as well as cause to increase in frequencies.

V. ACKNOWLEDGEMENT

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