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Mamdani fuzzy logic apparatus for solving classification problems problems

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The application of fuzzy logic theory in solving classification problems makes it possible to obtain fundamentally new models and methods for analyzing these systems. Based on the analysis carried out, it is proposed to apply the Mamdani algorithm as the main fuzzy inference method for constructing an optimal intelligent control model. The purpose of the presented work is to develop the Mamdani fuzzy model and use the matrix representation of fuzzy logic to solve classification problems.The problems faced by specialists in solving classification problems are identified: lack of information relevant in real time; restricting access to precise analysis; lack of databases of documented information and, as a result, there is no automated decision support system;lack of knowledge bases and rule bases for solving classification problems; lack of integrated software systems.Mamdani's fuzzy logic apparatus made it possible to improve the technologies for developing information systems.

I. INTRODUCTION

The problems faced by specialists in solving classification problems are identified:
- lack of information relevant in real time;
- restricting access to precise analysis;
- lack of databases of documented information and, as a result, there is no automated decision support system;
- lack of knowledge bases and rule bases for solving classification problems;
- lack of integrated software systems.

There are two types of uncertainty - stochastic (probabilistic) and linguistic (fuzzy). Stochastic uncertainty manifests itself in the form of the probability of a specific, well-defined event occurring. Linguistic uncertainty lies in the words of the statement that describe the phenomenon "approximately", inaccurately, indistinctly. In other words, with a probabilistic description, an event either completely occurs or does not occur at all, and with a fuzzy description, an event can occur "partially" [1].

When solving classification problems, the most widely used methods are: Mamdani, Tsukamoto, Larsen, Takagi-Sugeno. Below is a comparative analysis of fuzzy models [2-3]:

1) Mamdani models. The model can be defined as follows [4,5]:

The activation of sub conclusions in fuzzy rules is calculated by the formula, taking into account only active rules;

Accumulation of conclusions of fuzzy production rules, where fuzzy sets are combined and the final fuzzy set for the output linguistic variables is obtained;

In defuzzification, the result is brought to a crisp representation using the center of gravity method.

2) Tsukamoto model. Difference from the Mamdani model:

The algorithm is simple, but not universal, as it is used for monotonic matching functions of output parameters.

The accuracy is lower than the Mamdani algorithm.

3) Larsen model. Unlike the Mamdani model, it requires more multiplication operations.

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4) Takagi-Sugeno model. The main difference from the Mamdani model is setting the values of input variables in different ways that form a knowledge base: in the Mamdani algorithm, the values of output variables are set by terms, and in the Takagi-Sugeno algorithm, by a linear combination of input variables.

Based on the analysis carried out, it is proposed to use the Mamdani algorithm [6,7] as the main fuzzy inference method for constructing a classification model.

II. METHODS

The Mamdani algorithm in fuzzy inference systems is described as follows:

Suppose that the knowledge base is formed by two fuzzy rules [5-8]:

$$P_1 : if \;\; x \;\; is \;\; A_1 and \;\; is \;\; B_1 \;\; then \;\; z \;\; is \;\; C_1 \; P_2 : if \;\; x \;\; is \;\; A_2 and \;\; is \;\; B_2 \;\; then \;\; z \;\; is \;\; C_2$$

where x and y are the names of input variables, z is the name of the output variable, $A_1, A_2, B_1, B_2, C_1, C_2$ are some given membership functions, while a clear knowledge of $z_0$must be determined based on the information provided and clear knowledge of x_0, y_0.

This algorithm can be mathematically described as follows.

1. Fuzziness: find degrees of truth for the premises of each rule.

2. Fuzzy inference: cutoff levels are found for the prerequisites of each of the rules using the minimum operation.

3. Composition: using the max operation, the found truncated functions are combined, which leads to the final fuzzy subset for the output variable with the membership function

4. Reduction to clarity is carried out, for example, by the centroid method

Thus, a rule base was created using the Mamdani fuzzy logic apparatus.

The use of fuzzy logic theory for the analysis of business process management systems makes it possible to obtain fundamentally new models and methods for analyzing these systems [1-7].

1) Normalization

$U_{ij} = l\frac{x_{ij}-x_{minj}}{x_{maxj}-x_{minj}},\ l = 4.$

2) Bringing to fuzziness (process fuzzification)

$$\mu_{ij}^{k} = \frac{1}{1+\frac{(U_{ij}-k)^2}{\sigma^2}},$$
$$\mu_{ij}^{*} = \max_k \mu_{ij}^{k},\ l = 4k = 0, l.$$

3) Calculation of knowledge base terms

$$V_{ij} = \arg_k \mu_{ij}^{*}.$$

4) Fuzzy inference - Mamdani model.

5) Adequacy calculation

$$\mu_{ij} = \frac{1}{1+\frac{(U_{ij}-k)^2}{\sigma^2}}.$$

In the matrix representation [7], a two-dimensional vector x is introduced to describe the degree of truth of the statementA. If we introduce basis vectors $e^{(0)}$and $e^{(1)}$, then the vector x can be represented as

$$x = x_0 e^{(0)} + x_1 e^{(1)}\,, \tag{1}$$

moreover, the components of the vector x- numbers $x_0,\ x_1$- satisfy the conditions

$$0 \le x_0,\, x_1 \le 1,\, x_0 + x_1 = 1. \tag{2}$$

The basis vectors themselves $e^{(0)}$describe $e^{(1)}$statements with a degree of certainty of 0 and 1, respectively. The negation operation corresponds to the permutation of the vector components$x : \neg\, x = x_1 e^{(0)} + x_0 e^{(1)}$. In "classical" fuzzy logic, the assessment of the uncertainty of a statement causes difficulties [9]. Representing fuzzy statements in vector form allows us to calculate their uncertainty using the well-known information theory formula:

$$S(x) = -x_0 \log_2 x_0 - x_1 \log_2 x_1. \tag{3}$$

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Logical operations on vectors - conjunction, disjunction and implication - can be represented in matrix form. Let us compose from the components of the vector x conjunctive (), disjunctive $D\,()$and implicative $I\,()$matrices of size 2×2[7]:

$$C = \begin{pmatrix} 1 & x_0 \\ 0 & x_1 \end{pmatrix}; \begin{pmatrix} x_0 & 0 \\ x_1 & 1 \end{pmatrix}; \begin{pmatrix} x_1 & 0 \\ x_0 & 1 \end{pmatrix}; \tag{4}$$

Using these matrices, logical operations on fuzzy vectors can be represented as

$$x \wedge y = C(x)y;\ x \vee y = D(x)y;\ x \to y = D(x)y. \tag{5}$$

where the vector y should be thought of as a 1x2 matrix column composed of its components $y = \begin{pmatrix} y_0 \\ y_1 \end{pmatrix}$ and the dot denotes the usual matrix multiplication. The result of logical operations formulated in this form will also be a fuzzy vector. A detailed substantiation of the choice of matrices (3) is presented in [7]. Fuzzy predicate $x\,(s)$is a vector function defined on the definition set S, taking values in the space of fuzzy vectors F, whose elements are described by formulas (1) and (2), i.e.

$$x(s) = \begin{pmatrix} x_0(s) \\ x_1(s) \end{pmatrix} \tag{6}$$

$$0 \le x_0(s),\ x_1(s) \le 1;\ x_0(s) + x_1(s) = 1. \tag{7}$$

It follows from the definition of the predicate that the $x_1\,(s)$value of the membership function of the fuzzy variable is used as the value $\mu\,(s)$. While for the fulfillment of conditions (2) it is necessary $x_0\,(s) = 1\mu\,(s)$. Operations on fuzzy predicates are reduced to operations on fuzzy vectors [10-14].

In fuzzy logic applications, Mamdani 's algorithm is widely used, in which it is proposed to find the consequence of the rule y as a conjunction of the premise of the rule x and some variable w specified on the domain of definition y:

$$y = x \wedge w. \tag{8}$$

In matrix form, this expression looks like:

$$y = C\,(x)\,\mathrm{w}. \tag{9}$$

Fuzzy rule modus ponens can be written as follows through the implication:

$$x \to y = z. \tag{10}$$

In matrix form, this rule looks like:

$$I(x)y = z. \tag{11}$$

From formulas (9) and (11) the relationship between the variables w and z follows:

$$z = C(x)I(x)w. \tag{12}$$

C according to this formula, setting the variable **w**, as required by the Mamdani algorithm, we also set the fuzziness **z** of themodus ponens rule (10). The matrix method allows one to indicate the degree of fuzziness **z** of the rule (10).

III. RESULTS

Based on the matrix representation of Mamdani 's fuzzy logic on model tasks, classification results were obtained and a comparative analysis was carried out (Tables 1, 2 and 3).

Evaluation of the degrees of fuzziness of predicates allows, when solving specific problems, to carry out an additional choice of rules in such a way as to discard the least significant rules, and leave only the rules that carry the maximum information. In addition, with the help of such an estimate, it is possible to select truth functions that contain the maximum amount of information about the object under study.

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Table 1. Results obtained by the simple Mamdani model (%)

№	Bases taken for testing				Changes in the values of L						
	Name	N	M	Class count	3	5	7	9	11	13	15
1	Animal	101	16	7	100,0	100,0	100,0	100,0	100,0	100,0	100,0
2	Wine	178	13	3	98,31	100,0	100,0	100,0	100,0	100,0	100,0
3	Iris	150	4	3	67,33	96,00	97,33	98,00	98,67	100,0	100,0
4	Liver	178	10	2	80,34	90,45	96,63	98,88	99,44	99,44	99,44
5	Caesarian	80	5	2	57,50	57,50	57,50	57,50	57,50	57,50	57,50
6	Fire	244	10	2	86,07	93,85	96,72	99,18	99,18	100,0	100,0
7	Ecoli	336	7	8	66,96	87,50	93,45	97,02	98,81	99,70	100,0

Table 2. Results obtained through the matrix Mamdani model (%)

№	Bases taken for testing				Changes in the values of L						
	Name	N	M	Class count	3	5	7	9	11	13	15
1	Animal	101	16	7	100,0	100,0	100,0	100,0	100,0	100,0	100,0
2	Wine	178	13	3	100,0	100,0	100,0	100,0	100,0	100,0	100,0
3	Iris	150	4	3	85,33	98,00	98,67	99,33	99,33	100,0	100,0
4	Liver	178	10	2	86,52	94,94	97,75	98,88	99,44	100,0	99,44
5	Caesarian	80	5	2	57,50	57,50	57,50	57,50	57,50	57,50	57,50
6	Fire	244	10	2	96,31	97,54	100,0	100,0	100,0	100,0	100,0
7	Ecoli	336	7	8	77,38	89,88	94,05	96,43	98,81	100,0	100,0

IV. CONCLUSION

Thus, the degree of truth is recorded for each rule, since the number of expert assessments is high, one rule can be recorded for each assessment, while some may be contradictory or duplicated. To solve this problem and reduce the number of rules, for each rule we write down the degree of truth (the weight of the rule) and select the rule that has

Table 3. Differences in results

№	Name	N	M	Class count	3	5	7	9	11	13	15
1	Animal	101	16	7	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	Wine	178	13	3	1,69	0,00	0,00	0,00	0,00	0,00	0,00
3	Iris	150	4	3	18,00	2,00	1,34	1,33	0,66	0,00	0,00
4	Liver	178	10	2	6,18	4,49	1,12	0,00	0,00	0,56	0,00
5	Caesarian	80	5	2	0,00	0,00	0,00	0,00	0,00	0,00	0,00
6	Fire	244	10	2	10,24	3,69	3,28	0,82	0,82	0,00	0,00
7	Ecoli	336	7	8	10,42	2,38	0,60	-0,59	0,00	0,30	0,00

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a higher degree of truth.

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