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Magnetized particle motion around regular black holes immersed in an external uniform magnetic field

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In this work, we study magnetized particle motion around a magnetically charged regular Ayon-Beato Garcia black hole in an external uniform magnetic field. We analyse a chosen asymptotically uniform magnetic field, and we have plotted dependence of nonzero components of magnetic field on radial coordinate for the different values of mass of the black hole. We also extensively study magnetized particles motion around the black hole in an external uniform magnetic field, and we have found effective potential for chosen space-time. Using effective potential, magnetic coupling function is found. we have investigated center mass energy in vicinity BH for magnetized and charged particles. We also explored it for different values of the charge of the black hole.

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I. INTRODUCTION

In this work, we plan to extend these results to the charged magnetized particle motion and acceleration process around non-rotating regular black hole immersed in an external uniform magnetic field (without ergosphere) and study the influence of electric charge of black hole on these processes.

Particle motion around black holes is an important topic in astrophysics. Partly because if the particles are accelerated by black holes than black holes gain mass, and possibly black holes hurl particles away at high relativistic speeds due to the angular momentum barrier. The curvature singularity at the origin of compact objects is formed as a result of a gravitational collapse of the end state of the evolution of massive stars. It is the one of the main fundamental problems of the general relativity being the classical theory. One of the first attempts to solve this issue has been proposed by Bardeen. In their pioneering work, it was suggested that the regular black hole solution describing the nonsingular compact object surrounded with the event horizon satisfies the weak energy conditions. The solution obtained by Bardeen is not a vacuum solution of the Einstein equations, and its physical interpretation has been delayed for decades. The physical interpretation of this type of solutions has been proposed in the re-obtained solution of regular black hole by Ayon-Beato and Garcia [1–3]. They have obtained the regular black hole solution introducing the nonlinear magnetic monopole charge of the black hole, which affects the gravitational field of the latter. The thermodynamic properties of Bardeen's solution have been investigated in [4].

Einstein's theory of gravity has been tested in both weak and strong field regimes using Solar system tests and observations of the shadow of a supermassive black hole [5, 6] at the center of elliptical galaxy M87. The magnetized particles motion around non-rotating and rotating black holes in the presence of an external magnetic field have been

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studied in [7, 8]. Further developments of the magnetized particles motion around a black hole in different gravity models can be found in [9–12].

The observational evidences [13] indicate that the magnetic field arises due to plasma which may exist in the surrounding of black holes in the form of an accretion disk or a charged gas cloud [14]. The relativistic motion of charged particles in the accretion disk generates the magnetic field. This magnetic field also leads to the generation of gigantic jets along the magnetic axes. Since GR predicts that all kinds of fields produce gravitational effects via curvature of space-time, it is assumed that the magnetic field is relativity weak such that it does not affect the geometry of the black hole yet it may affect the motion of charged particles[15]. The motion of charged particles is chaotic and unpredictable for strong electromagnetic and gravitational fields. Moreover the corresponding orbits are unstable except the innermost stable circular(ISCO) orbit.

If the collision of two particles occurs near the event horizon of black hole than high center of mass energy is produced. Bañados, Silk and West (BSW) [16] have proposed the mechanism for the collision of two particles falling from rest at infinity towards the Kerr black hole and determined that the center of mass (CM) energy in the equatorial plane may be highest in the case of an extremal black hole or the fast rotating black hole. Lake [17] found that the CM energy of the particles at the inner horizon of Kerr black hole. Further, Wei et al. [18] investigated that the CM energy of the collision of the particles around Kerr-Newmann black hole. A general review of the collision mechanism is found in [19].

Throughout this work we use signature $(-,+,+,+)$ for the space-time and geometrized unit system $G = c = 1$ (However, for an astrophysical application we have written the speed of light explicitly in our expressions). Latin indices run from 1 to 3 and Greek ones from 0 to 3.

II. BREFLY ABG FIELD

In this section we study electromagnetic four potentials
The line element of non rotating ABG regular black holes is given in.. as

$$ds^2 = -f dt^2 + f^{-1} dr^2 + d\Omega^2 \tag{1}$$

where

$$f = 1 - \frac{2Mr^2}{(r^2+q^2)^{3/2}} + \frac{q^2 r^2}{(q^2+r^2)^2} \tag{2}$$

the associated electric field E_r is given by

$$E_r = qr^4\Big[\frac{r^2-5q^2}{(r^2+q^2)^4} + \frac{15}{2}\frac{M}{(r^2+q^2)^{7/2}}\Big]$$

One can find

$$A_t = \int E_r dr = \frac{qr^5}{2}\left[\frac{3M}{q^2\left(q^2+r^2\right)^{5/2}} - \frac{2}{\left(q^2+r^2\right)^3}\right] + C$$

here C is integral constant. One can find the constant using limit when $q \to 0$ scalar electric potential also should be zero.

$$\lim_{q\to 0}\frac{qr^5}{2}\left(\frac{3M}{q^2\left(q^2+r^2\right)^{5/2}} - \frac{2}{\left(q^2+r^2\right)^3}\right) + C = \frac{3M}{2q} + C$$

then $C = -\frac{3M}{2q}$

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A. Electromagnetic field around ABG in magnetic field

in this subsection we consider the RBH modelled by ABG immersed in an asymptotically uniform magnetic field. The electromagnetic four-potential can be found using Wald solution,

$$A_\phi = \frac{1}{2}Br^2\sin\theta, \tag{3}$$

$$A_t = \frac{1}{2}\left[\frac{3Mr^5}{q\left(q^2+r^2\right)^{\frac{5}{2}}} - \frac{2qr^5}{\left(q^2+r^2\right)^3}\right] - \frac{3M}{2q}$$

$$= \left[\frac{3M}{2q}\frac{\sqrt{r^2+q^2}}{r} - \frac{q}{r}\right]\left(\frac{r^2}{r^2+q^2}\right)^3 - \frac{3M}{2q} \tag{4}$$

and non zero components of the electromagnetic tensor ($F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$) is

$$F_{r\phi} = Br\sin^2\theta \tag{5}$$

$$F_{rt} = \frac{qr^4}{(r^2+q^2)^3}\left[\frac{r^2-5q^2}{r^2+q^2} + \frac{15}{2}\frac{M}{(r^2+q^2)^{\frac{1}{2}}}\right] \tag{6}$$

$$F_{\theta\phi} = Br^2\sin\theta\cos\theta \tag{7}$$

nonzero components of magnetic field

$$B^{\hat{r}} = B\cos\theta, \qquad B^{\hat{\theta}} = \sqrt{f(r)}B\sin\theta \tag{8}$$

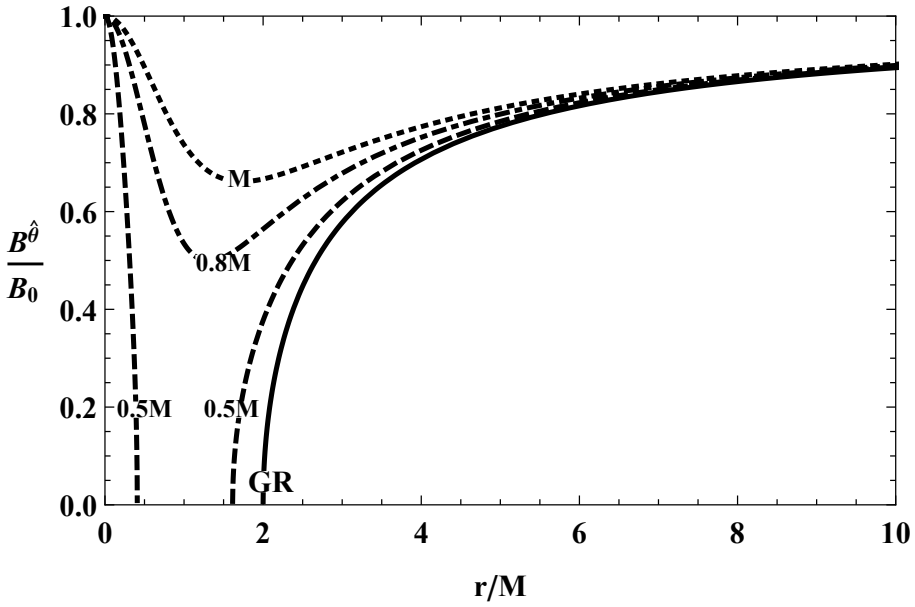


FIG. 1: Radial profile of $B^{\hat{\theta}}$

III. MAGNETIZED PARTICLE MOTION IN SPHERICALLY SYMMETRIC SPACETIME

H-J equation for magnetized particle is written in ... in the following form

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu} = -\left(m - \frac{1}{2}D^{\mu\nu}F_{\mu\nu}\right)^2 \tag{9}$$

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where $D^{\mu\nu}F_{\mu\nu}$ is responsible to interaction between magnetized particle and magnetic field According to. de Filice 2003 $D^{\alpha\beta}$ is

$$D^{\alpha\beta} = \eta^{\alpha\beta\sigma\nu}u_\sigma\mu_\nu \qquad D^{\alpha\beta}u_\beta = 0 \tag{10}$$

here μ^ν is the magnetic dipole moment four-vector and u^ν four-velocities of the particle, Electromagnetic field tensor $F_{\alpha\beta}$ can be decomposed through electric E_α and magnetic B^α fields in the following form

$$F_{\alpha\beta} = u_{[\alpha}E_{\beta]} - \eta_{\alpha\beta\sigma\gamma}u^\sigma B^\gamma \tag{11}$$

taking into account above equation ... one may calculate the interaction term $D * F$ in the following form

$$D^{\mu\nu}F_{\mu\nu} = 2\mu^\alpha B_\alpha = 2\mu B_0\mathcal{L}[\lambda_{\hat{\alpha}}] \tag{12}$$

where μ is module dipolar magnetic magnetic moment of the particle and $L[\lambda_{\hat{\alpha}}]$ is a somewhat complicated function of the spacetime coordinates, as well as of other parameters defining the tetrad $\lambda_{\hat{\alpha}}$ attached to the comoving fiducial observer (e.g. the orbital angular velocity of the particle).

Now we plan to study orbital motion of magnetized particles around the regular black hole in the $\left(D^{\mu\nu}F_{\mu\nu}\right)^2 \to 0$ limit. It is clear that at the equatorial plane i.e. $\theta = \pi/2$, θ components of the four-momentum $p_\theta = 0$, but other components are conserved. The spacetime symmetries are preserved by the axial symmetric configuration of the magnetic field and, therefore, they still allow for two conserved quantities: $p_\phi = L$ and $p_t = -E$. These are energy and angular momentum of particle, respectively. From (6) using (9) one can get the expression

$$\dot{r}^2 = \mathcal{E}^2 - 1 - 2V_{eff}(r, q, l, \beta) \tag{13}$$

the effective potential

$$V_{\text{eff}}(r, q, l, \beta) = \frac{1}{2}\Big[f\left(1 + \frac{l^2}{r^2} - \beta\mathcal{L}[\lambda_{\hat{\alpha}}]\right) - 1\Big] \tag{14}$$

where $\beta = 2\mu B_0/m$ magnetic coupling parameter now we will calculate circular orbits with the standard way

$$\dot{r} = 0 \qquad \frac{\partial V_{\text{eff}}}{\partial r} = 0 \tag{15}$$

Using eff

$$\beta(r, l, \mathcal{E}, q) = \frac{1}{\mathcal{L}[\lambda_{\hat{\alpha}}]}\Big(1 + \frac{l^2}{r^2} - \frac{\mathcal{E}^2}{f}\Big) \tag{16}$$

from eff and circular condition

$$D * F = 2\mu B_0 f\, e^\Psi \tag{17}$$

comparing ....

$$\mathcal{L}[\lambda_{\hat{\alpha}}] = e^\Psi f \tag{18}$$

then taking into consideration

$$\frac{\partial V_{\text{eff}}}{\partial r} = f\mathcal{L}[\lambda_{\hat{\alpha}}]\frac{\partial\beta}{\partial r} \tag{19}$$

$$e^\Psi = (f - \Omega^2 r^2)^{-1/2} \tag{20}$$

where Ω is angular momentum of the particle then the magnetic interaction term takes following form

$$\Omega = \frac{d\phi}{dt} = \frac{d\phi/d\tau}{dt/d\tau} = -\frac{f}{r^2}\frac{l}{\mathcal{E}} \tag{21}$$

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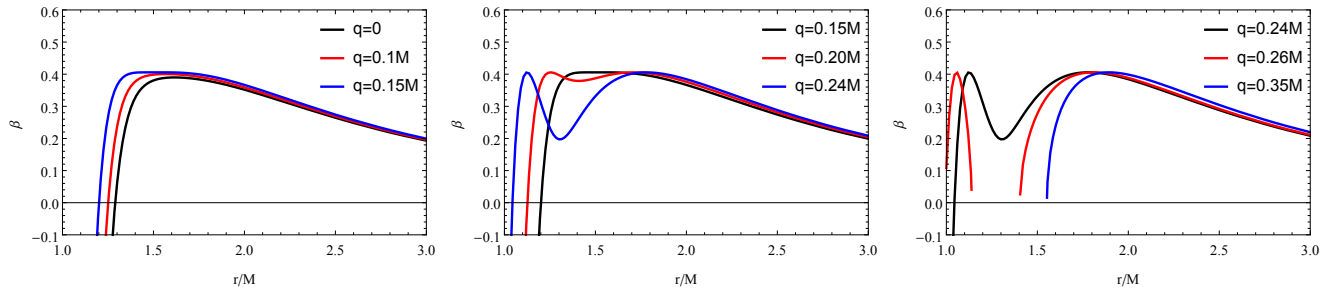


FIG. 2: The dependence of magnetic coupling function from the radial coordinate for the different values of the black hole charge

magnetic coupling parameter $\beta(r,l,\mathcal{E},q)$

$$\beta(r,l,\mathcal{E},q)=\left(\frac{1}{f}-\frac{l^2}{\mathcal{E}^2r^2}\right)^{1/2}\left(1+\frac{l^2}{r^2}-\frac{\mathcal{E}^2}{f}\right) \tag{22}$$

stable orbits of the magnetized particles can be find using following set of equations:

$$\beta=\beta(r,l,\mathcal{E},q),\qquad \frac{\partial\beta(r,l,\mathcal{E},q)}{\partial r}=0 \tag{23}$$

This is a system of two equations with five unknown quantities $\beta,r,l,\mathcal{E},q$ and black hole charge q, so its solution can be parameterized in terms of any two of the five independent variables. We will use the magnetic coupling and the orbital radius r as free parameters. Our aim is then to find the angular momentum l and the (specific) energy $\mathcal{E}$ of the particle as functions

$$\mathcal{E}^2_{\rm min}=\frac{f}{6f_1r^2}\Big[fl^2+f_1\left(1-\frac{l^2}{r^2}\right) -\sqrt{f^2l^2+2ff_1l^2\left(17l^2+7r^2\right)+f_1^2\left(1-\frac{l^2}{r^2}\right)^2}\Big] \tag{24}$$

where

$$f_1=\frac{3r^2}{q^2+r^2}\left(1-f-\frac{Mr^2}{\left(q^2+r^2\right)^{3/2}}\right)+f-1$$

substituting (24) to (22) we have

$$\beta_{\rm min}=\frac{(5f_1+f_2)r^2-(f-7f_1)\,l^2}{6\sqrt{f}f_1r^2}\sqrt{1-\frac{6f_1l^2}{(f_1-f)\,l^2-(f_1-f_2)r^2}} \tag{25}$$

where

$$f_2^2=al^4+bl^2+f_1^2r^4,\quad a=f_1\left(34f+f_1\right)\ ,\quad b=f^2+2\left(7f-f_1\right)f_1r^2 \tag{26}$$

IV. MAGNETIZED PARTICLES' ACCELERATION IN REGULAR BLACK HOLE IMMERSED IN AN EXTERNAL MAGNETIC FIELD

Here we investigate energy extraction from black hole using collision of two particles around (at the circular orbits) a regular black hole immersed in magnetic field. we study the effect of black hole charge and external magnetic field to the center-mass-energy of the two colliding particles which is coming from infinity with energies $\mathcal{E}_1$ and $\mathcal{E}_2$. Using well known expression for center of mass energy

$$\mathcal{E}^2_{cm}=\frac{E^2_{cm}}{2m_0c^2}=1-g_{\alpha\beta}v_1^\alpha v_2^\beta \tag{27}$$

here $g_{\alpha\beta}$ is metric tensor of the spacetime.v_1^α and v_2^β are 4-velocities of the colliding particles

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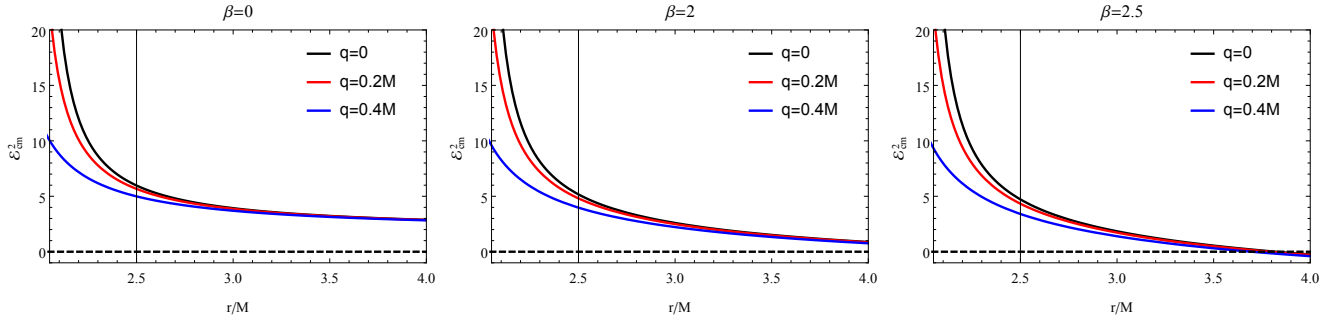


FIG. 3: Radial dependence of the center of mass energy in the different values of black hole charge and magnetic field. Here we consider head on collision taking values for the angular moments $l_1 = 2$ and $l_2 = -2$

A. Collision of two magnetized particles

The four-velocities of the magnetized particle IN circular orbits at equatorial plane ($\dot{\theta} = 0$)

$$\begin{aligned} \dot{t} &= -\frac{\mathcal{E}}{f(r)} \\ \dot{r} &= \sqrt{\mathcal{E}^2 - f(r)(1 + \frac{l^2}{r^2} - \beta^2)} \\ \dot{\phi} &= \frac{l}{r^2} \end{aligned} \tag{28}$$

and for neutral particles

$$\begin{aligned} \dot{t} &= -\frac{\mathcal{E}}{f(r)} \\ \dot{r} &= \sqrt{\mathcal{E}^2 - f(r)(1 + \frac{l^2}{r^2})} \\ \dot{\phi} &= \frac{l}{r^2} \end{aligned} \tag{29}$$

the expression for center of mass energy of the two magnetized particles

$$\mathcal{E}^2_{cm} = 1 + \frac{\mathcal{E}_1\mathcal{E}_2}{f(r)} - \frac{l_1 l_2}{r^2} - -\sqrt{\mathcal{E}_1^2 - f(r)(1 + \frac{l_1^2}{r^2} - \beta_1^2)} \times \sqrt{\mathcal{E}_2^2 - f(r)(1 + \frac{l_2^2}{r^2} - \beta_2^2)} \tag{30}$$

B. Collision of two magnetized and charged particles

four-velocities of charged particle can be find using The Lagrangian for the charged particle in curved space influence of the electromagnetic field

$$\mathcal{L} = m g_{\mu\nu} u^\mu u^\nu + e u^\mu A_\mu \tag{31}$$

The conserved quantities, the energy and the angular momentum can be found by

$$p_t = \frac{\partial \mathcal{L}}{\partial \dot{t}} = m g_{tt} \dot{t} + e A_t, \tag{32}$$

$$p_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = m g_{\phi\phi} \dot{\phi} + e A_\phi, \tag{33}$$

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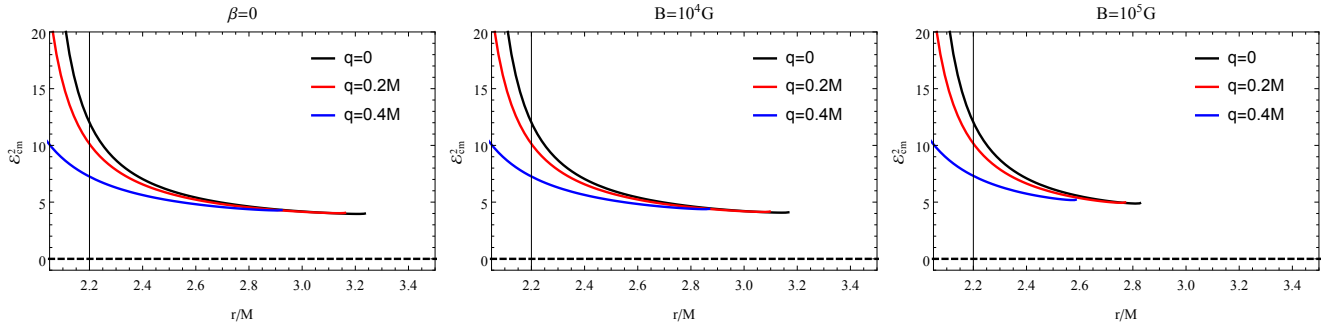


FIG. 4: Radial dependence of the center of mass energy in the different values of black hole charge and magnetic field. Here we consider head on collision taking values for the angular moments $l_1 = 2$ and $l_2 = -2$

and four-velocities of the charged particle

$$\begin{aligned} \dot{t} &= \frac{1}{f(r)}(\mathcal{E} - \frac{Q}{r}) \\ \dot{r} &= \sqrt{\left(\mathcal{E} - \frac{Q}{r}\right)^2 - f(r)\left[1 + \left(\frac{l}{r} - \frac{1}{2}\omega_B r\right)^2\right]} \\ \dot{\phi} &= \frac{l}{r^2} - \frac{1}{2}\omega_B \end{aligned} \tag{34}$$

the expression for center of mass energy of the two magnetized particles

$$\mathcal{E}^2_{cm} = 1 + \frac{\mathcal{E}_1\mathcal{E}_2}{f(r)} - \frac{l_1 l_2}{r^2} - -\sqrt{\left(\mathcal{E}_1 - \frac{Q}{r}\right)^2 - f(r)\left[1 + \left(\frac{l_1}{r} - \frac{1}{2}\omega_B r\right)^2\right]}\sqrt{\mathcal{E}_2^2 - f(r)(1 + \frac{l_2^2}{r^2} - \beta^2)}$$

C. Collision of two charged particles

The expression for center of mass energy of the two magnetized particles

$$\mathcal{E}^2_{cm} = 1 + \frac{1}{f(r)}\left(\mathcal{E}_1 - \frac{Q}{r}\right)\left(\mathcal{E}_2 - \frac{Q}{r}\right) - \frac{l_1 l_2}{r^2} - -\sqrt{\left(\mathcal{E}_1 - \frac{Q}{r}\right)^2 - f(r)\left[1 + \left(\frac{l_1}{r} - \frac{1}{2}\omega_B r\right)^2\right]} \times \sqrt{\left(\mathcal{E}_2 - \frac{Q}{r}\right)^2 - f(r)\left[1 + \left(\frac{l_2}{r} - \frac{1}{2}\omega_B r\right)^2\right]} \tag{35}$$

V. CONCLUSION

In this work, we have studied magnetized particle motion around a regular black hole in an external uniform magnetic field. Moreover, we have discussed center-mass-energy for various types of particles. From the performed research we summarize our main results as:

- We have analyzed a chosen asymptotically uniform magnetic field, and we have plotted dependence of nonzero components of magnetic field on radial coordinate for the different values of mass of BH in Fig.1.
- We have extensively studied magnetized particle motion around BH in an external uniform magnetic field, and we have found effective potential for chosen space-time. Using effective potential, magnetic coupling function is found, and the obtained results are represented in Fig.2.
- Finally, we have investigated center mass energy in vicinity BH for magnetized and charged particles. We explored it for different values of the charge of BH and plotted the dependence of the energy of the center of mass on the radial coordinate Fig.3 and 4.

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VI. ACKNOWLEDGEMENT

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