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Pre-processing and Estimation of Image Quality Based on the Apparatus of fuzzy set

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The issues related to pre-processing of images based on the apparatus of fuzzy sets are considered. The architecture of the image processing system is proposed, as well as the main stages of digital image processing using the apparatus of fuzzy sets and materials on the formation of the rule database. The issues of improving the image brightness and contrast using the apparatus of fuzzy sets are considered. An algorithm for quantifying image quality by fuzzy logic methods is proposed.

Keywords: Image; brightness; contrast; noise; fuzzy set; fuzzy logic; quality; estimation

I. INTRODUCTION

It is known that data processing systems (DPS), developed in recent years, are char-acterized by a high level of use of various algorithms and technologies for the intellectual-ization of data processing operations. One of the main directions in the field of intellectu-alization of DPS is information processing systems presented in the form of digital images.

Technologies for the intellectualization of image processing operations are in de-mand when creating video surveillance and access control systems. Such systems are based on methods and algorithms related to the issues of biometric identification of a person [43–47]. In applied systems for image analysis and processing, the main functional tasks are image quality assessment, object edge detection, classification, clustering, and pattern recognition.

The main methodological concepts in image processing have long been based on various modifications of spectral methods, and the difficulties in solving applied prob-lems were attributed to the shortcomings of equipment and hardware – memory shortage and low speed. At present, scientific and technological progress has led to a sharp in-crease in the capabilities of computer technology and the construction of specialized devices on its basis, which, it would seem, should have led to the solution of the main prob-lems of image perception by technological systems. However, the problem has not disap-peared; the understanding of its significance and specific difficulties increased. The cur-rent situation is forcing researchers to place an emphasis on the development of new ap-proaches and methods in image processing in order to find ways to bridge the gap be-tween "machine" and "human" perception.

In recent years, the use of fuzzy methods in the problems of image analysis and pro-cessing has been developing more intensively [48–55], this is associated with the following: 1) these methods are powerful tools for knowledge representing and processing; 2) these methods can effectively control uncertainty and ambiguity. In the theory of fuzzy set and fuzzy logic, there are powerful tools for the representation and process of human cogni-tion in the form of fuzzy IF-THEN rules [56]. Fuzzy rules are constructed from the mem-bership function of inputs and outputs [56–58]. Inputs and outputs reduced to a fuzzy form facilitate the selection of rules and their generalization. In general, the rules can be derived from the experience of one or more experts in the subject area under consideration. Undoubtedly, the definition of good linguistic rules depends on the amount and quality of the expert's knowledge.

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However, at present, there is no formalized approach to the prob-lem of accurate and unambiguous representation of knowledge in the framework of fuzzy set theory [59–61]. In this regard, the representation of knowledge in a fuzzy form is arbi-trary.

This article discusses the problems of enhancement and assessment of image quality.

The purpose of this study is to develop algorithms for noise suppression by averag-ing filters and to evaluate image quality.

To achieve these goals, it is necessary to solve the following tasks:

1) to analyze the status of image enhancement issues and formulate research objec-tives;

2) to develop algorithms for enhancement and assessment of image quality based on the theory of fuzzy sets;

3) to conduct experimental studies to test the performance of the developed algorithms.

II. LITERATURE REVIEW

This section discusses the state-of-the-art of image processing using the fuzzy set concept in relation to the problem of enhancement of image quality. The quality of a halftone image depends on many parameters, the main ones being [23, 62–64]:

- the brightness of an image pixel is a characteristic that determines its measure of difference from black;

- contrast can be quantified as the difference between the maximum and minimum brightness values of an image.In other words, it is a measure of how wide is the spread in the colors of the image pixels;

- noise can be quantified as the arithmetic mean of the spread of brightness values in an image.

The main shortcomings of images, in most cases, are distorted brightness character-istics, noise and low contrast.

Therefore, the main goal of enhancement methods is to transform images to such a form, which makes them more contrasting and, accordingly, more informative [24–26].

One of the main and most intensively developing areas of applied mathematics and computer science is fuzzy set theory and fuzzy logic. This is due to the fact that in recent years the use of these methods has been expanding in solving various applied problems. With their help, many applied problems of data processing are solved, in particular, problems of analysis and image processing. Therefore, an ever wider circle of specialists pays attention to the problem of digital image processing using fuzzy set theory and fuzzy logic. This circumstance is due to the fact that in many image processing applications it is required to use expert knowledge to overcome some difficulties (e.g., object recognition, scene analysis) [27, 28]. As already noted, fuzzy set theory and fuzzy logic have powerful tools for representing expert knowledge in the form of fuzzy IF-THEN rules. On the other hand, many of the difficulties in image processing arise from the randomness, ambiguity, and uncertainty in the data used in the tasks under consideration. To work with random-ness in image processing, probability theory can be used; for other types of imperfections, geometric fuzziness can be represented on the basis of the apparatus of fuzzy sets and fuzzy logic [26–31].

The study given in [26] was one of the first studies, where the issues of quality en-hancement were considered using the apparatus of fuzzy logic. It describes a method for extracting the fuzzy properties of a grayscale image, which can be used to increase the contrast of this image. Quality enhancement of original image is usually one of the first steps in computer vision tasks. Image enhancement techniques typically remove noise, smooth out regions where gray levels do not change significantly, and emphasize sharp changes in gray levels.

Since the fuzzy logic apparatus allows incorporating heuristic knowledge about its particular application in the form of rules, it ideally suits for building an image enhance-ment system. This has led to the development of various image enhancement methods based on fuzzy logic. Below we will briefly consider some of them.

In [27], a filter was proposed to dynamically reduce the narrowing of the range of brightness values and increase the contrast, using an approach based on fuzzy rules. The method is based on the algorithm given in [28]. In [29], a nonlinear fuzzy filter for image processing was proposed. It is known that averaging filters effectively remove Gaussian noise, and filters based on order statistics, such as the median filter, are effectively used to remove impulse noise. Fuzzy logic was used in [30] to combine these two filters.

Despite the progress made in the field of digital image processing using the appa-ratus of fuzzy sets, there are a number of unsolved problems. These include the problem of adequate mapping of the object domain to a fuzzy system, the choice of fuzzy inference models and their integration into a single intelligent system [28–33].

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III. FORMULATION OF THE PROBLEM

Consider image F of size $M \times N$ pixel, having a gray level r between 0 and $L-1$. When using a fuzzy set for image processing, it is considered as an array of fuzzy singletons. Each element of the array points to the membership value $\mu_F(f(x,y))$ of gray level $f(x,y)$ corresponding to the (x,y)-th pixel, in accordance with the predefined image properties, such as brightness, sharpness, uniformity [26–28].

As a generalization of this approach, we introduce the following image representation in a fuzzy environment.

Image F, described in a fuzzy environment, has the following form

$$F = \{\langle f(x,y),\ \mu_F(f(x,y))\rangle \,|f(x,y) \in \{0,...,L-1\}\},$$

where $x \in \{1,...,M\}$, $y \in \{1,...,N\}$, $\mu_F(f(x,y))$ denote the membership degree of the (x,y) -th pixel to the set in accordance with the properties of the image.

The knowledge base consists of a certain set of fuzzy rules. Generally, the number of rules required in a system is related to the number of controlled parameters. For example, fuzzy rules for smoothing sharpness and highlighting contours in a grayscale image, which consist of three inputs and the same number of outputs [34, 35] are:

IF a pixel is darker than neighboring pixels
THEN to make it lighter
OTHERWISE
IF a pixel is lighter than neighboring pixels
THEN to make it darker
OTHERWISE to leave it unchanged.

In these rules, the differences in brightness values between the pixel in question and its neighbors are the input data, and the increase and decrease in the brightness values of that pixel are the output data. If we assume that the image brightness values are in the in-terval [0, L-1] (where L is the number of gradations of brightness), then simple triangular fuzzy sets - the average positive and the average negative are defined in the interval [-L+1, L-1] and represent the bright and dark brightness values of the input variables, and the small positive, zero, and small negative triangular fuzzy sets are defined as the incre-ments of the specified value according to the compiled rules. The output value is added to the original pixel brightness value.

Rules or fuzzy associations are knowledge that is important for the system in terms of response to all possible combinations of inputs. Fuzzy systems remember sets of fuzzy associations or rules [34, 35].

To quantify these parameters, the main input variables are the factors that affect the image quality. The very quality of the processed image is the sought-for output variable. Let the fuzzy variable be described by the set F—a universal set (area of reasoning), A—fuzzy set on F. The rule database containing fuzzy statements in the form "IF-THEN" and membership functions for the corresponding linguistic terms act as the basis for car-rying out the fuzzy inference. At that, there must be at least one rule for each linguistic term of the output variable, and any term of the input variable has at least one rule in which this term is used as a prerequisite. Otherwise, the base of fuzzy rules is incomplete [24–28].

Functions μ_F correspond to the membership of a set of image components. Images fed into a computer often turn out to have small changes in brightness compared to their average values. In this case, the brightness does not change from black to white, but from gray to a slightly lighter gray. That is, the actual range of brightness is much less than the allowable one (grayscale). The task of increasing the contrast is to "stretch" the range of the image brightness to the full scale. Image processing techniques based on the fuzzy set theory provide a flexible mathematical framework to deal with "qualitative" properties such as image contrast in the presence of ambiguity and fuzziness often found in digital images [26]. For this purpose, in [36, 37], an algorithm for increasing the contrast and brightness of images in a fuzzy environment was proposed. The result of image processing by the contrast enhancement algorithm is denoted by $g_1(x,y)$, and by the brightness enhancement algorithm is denoted by $g_2(x,y)$. Consider the elimination of the third factor - noise in a fuzzy environment. For comparison, consider the result of image processing by the arithmetic mean and geometric mean filters.

IV. NOISE SUPPRESSION BY AVERAGING FILTERS

Consider a noisy image $z(x,y)$ formed by adding noise $\eta(x,y)$ to the original image $f(x,y)$:
$z(x,y) = f(x,y) + \eta(x,y)$.

Let S_{xy} denote a rectangular neighborhood, i.e. some m×n image area centered on a pixel (x, y). The brightness value of a pixel with coordinates (x, y) of the reconstructed image is this value [58, 61]:

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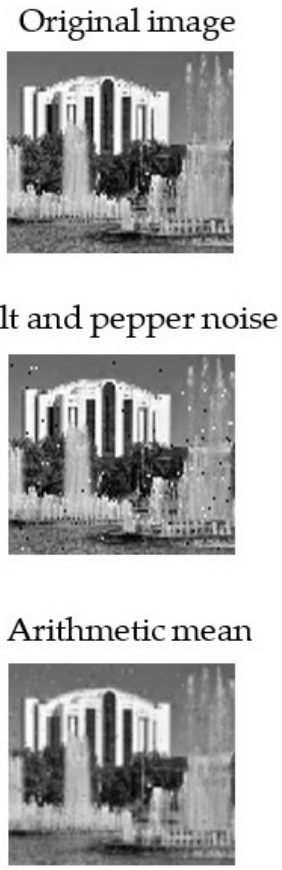


FIG. 1. Arithmetic mean for local window size 3x3

$$g(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} z(s,t)$$

If on the image processed by the arithmetic mean filter with an aperture of 3×3 (IV), regular structures are still distinguishable, then after processing with a filter with an aperture of 9×9 (IV), they are invisible, despite significant brightness differences in them.

The result of image processing by the geometric mean filter is described by the expression [58–61]

$$g\,(x,y) = \left[\prod_{(s,t)\in S_{xy}} z(s,t) \right]^{\frac{1}{mn}} \tag{1}$$

Here, the value of the reconstructed image in each pixel is the root of the mn power of the product of the brightness values in the pixels of the neighborhood S_{xy}.

Based on expression (1), it can be assumed that the result of processing will signifi-cantly depend on the size of the local aperture. This is confirmed by the images processed by this method, shown in Figures 3 and 4.

Anti-aliasing filters, both linear and non-linear ones, have a common disadvantage: by suppressing the upper frequency components of the image, they not only reduce the ef-fect of noise, but lead to the loss of small objects in the image and many tonal transitions that are not step-typed. In addition, the procedure for arranging pixels within a window requires significant computational costs even for small images. Standard windowed fil-ters are focused on the constant characteristics of the reference signal; they cannot take in-to account the changing local characteristics of the images. Their effect does not depend on the nature of the image: its texture or a smooth color area. To solve this problem, a fuzzy approach is used directly to the processing of window pixels.

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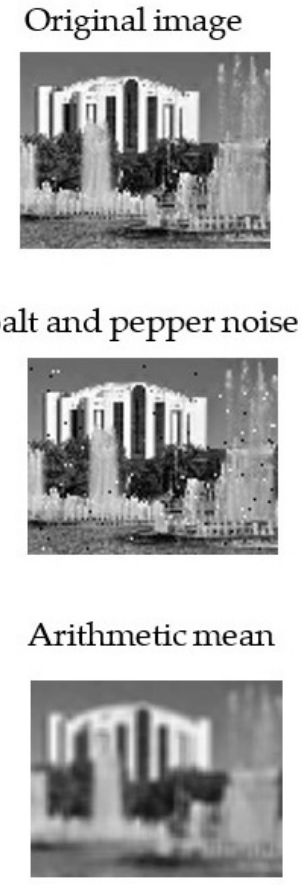


FIG. 2. Arithmetic mean for local window size 9x9

Consider a fuzzy set $A = \{(z, \mu_A(z))\}$, where Z is a discrete set of elements $\{z_1, ..., z_n\}$. When considering discrete fuzzy sets and random events, it should be taken into account that the membership function is set only on a discrete set of elements from Z or R^n. Below is an expression for fuzzy image processing:

$$g_3(x,y) = \frac{\sum_{i=1}^{n} \sum_{(s,t)\in S_{xy}} \mu(z_i) z_i(s,t)}{\sum_{i=1}^{n} \mu(z_i)}$$

Figure 5 shows fuzzy processing at local window sizes 3×3 and 9×9.

When creating a fuzzy model for quantifying image quality and training parameters with neural networks, the main factors that affect image quality were taken as input vari-ables.

1. Brightness is a characteristic that determines how much pixel colors differ from black.
2. Contrast can be defined as a range of tones of average brightness.

With an increase in contrast, the range of brightness of mid-tones narrows, and with a decrease in contrast, on the contrary, it increases, so with an increase in contrast, pixels that have a brightness above a certain value become even brighter, and pixels that have a brightness below a certain value become darker.

3. Noise is a random error at each point of the image.

V. NEURO-FUZZY NETWORK FOR QUANTIFYING IMAGE QUALITY

When constructing a neuro-fuzzy network for quantitative assessment of image qual-ity, three terms are used for the linguistic assessment of each variable - "bad", "good", "ex-cellent". For the output variable, which denotes image quality, we use five terms (bad, worse, good, better, excellent) with Gaussian membership functions. When drawing

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FIG. 3. Geometric mean for local window size 3x3



FIG. 4. Geometric mean for local window size 9x9

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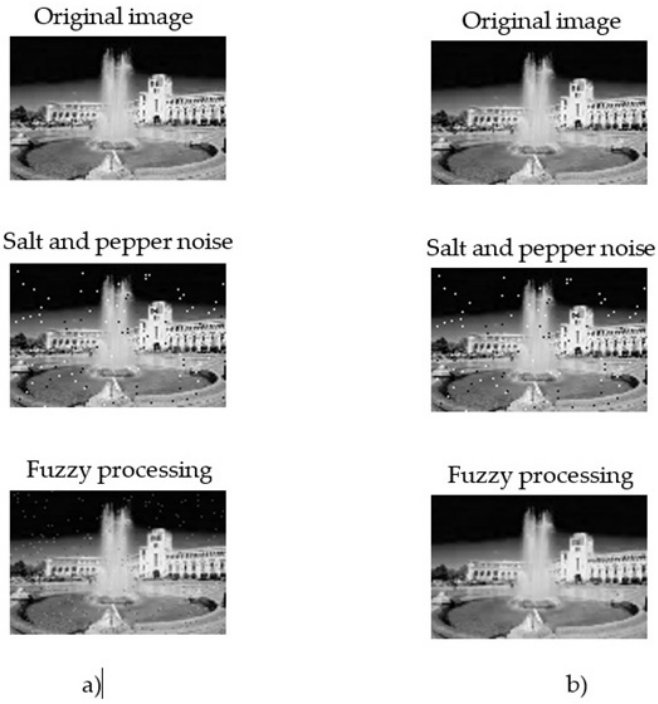


FIG. 5. Fuzzy processing at local window sizes: a-3x3; b-9x9

up the logical rules, it is taken into account that the image quality deteriorates in the presence of noise, poor brightness and contrast. When these factors are absent or reduced, the image quality improves. The image quality assessment model is based on Sugeno's fuzzy infer-ence, in which all values of input and output variables are given by fuzzy sets. The Sugeno knowledge base can be interpreted as a partition of the space of influencing factors into subdomains with fuzzy boundaries, within which the response function takes on a fuzzy value.

The neuro-fuzzy network for quantifying the image quality of the original image and the image obtained after processing consists of six layers and a zero layer, which receives the input signal in the form of a matrix, the columns of which correspond to different classes of the image being processed. In zero layer, this signal is transformed into a vector with elementsq_i^k, where$i = \overline{1, N \times M}$, in our prob-lem, k takes values from zero to 3 i.e. $q_i^0 = f$,$q_i^1 = g_1$, $q_i^2 = g_2$, $q_i^3 = g_3$respectively, and $N \times M$ determines the number of simultaneously processed pixels [61].

1.In the first layer, each component of the input signal q_i^k is fuzzified using a set of membership functions $\mu_{ji}^k(q_i^k)$, $j = 1, 2, ..., m$, in our case, they are the Gaussian functions of the following form

$$\mu_{ji}^k(q_i^k) = \exp\left(-\frac{1}{2}\left(\frac{q_i^k - c_j^k}{\sigma_j^k}\right)\right)$$

where c_j^k determines the position of the center of the j-th membership func-tion of the k-th class, and σ_j^kdetermines its width.

2. In the second hidden layer, the membership levels of the input signals are aggregated for each class based on the calculation of the fuzzy T-norm of the following form

$$w_j^k = \prod_{l=1}^{N \times M} \mu_{jl}^k(q_i^k).$$

3. In the third layer, normalization is implemented:

$$\bar{w}_j^k = \frac{w_j^k}{\sum\limits_{j=1}^{m} \prod\limits_{l=1}^{N \times M} \mu_{jl}^k(q_i^k)},$$

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where

$$\sum_{j=1}^{m} \bar{w}_j^k = 1.$$

4. In the fourth hidden layer, m functions of the consequent are calculated

$$r_j^k(q^k) = b_{j0}^k + \sum_{l=1}^{N\times M} b_{jl}^k q_l^k, \tag{2}$$

and they are multiplied by $\bar{w}_j^k$, calculated in the third layer, in the form $\bar{w}_j^k r_j^k(q^k)$.

5. In the fifth hidden layer, the following estimates are calculated

$$\tilde{q}^k = \sum_{j=1}^{m} \bar{w}_j^k r_j^k(q^k) = \sum_{j=1}^{m} \frac{\prod\limits_{l=1}^{N\times M} \mu_{jl}^k(q_i^k)}{\sum\limits_{j=1}^{m} \prod\limits_{l=1}^{N\times M} \mu_{jl}^k(q_i^k)} r_j^k(q^k), \tag{3}$$

6. In the output sixth layer, the output $\tilde{q}^k$ of the processed image is formed. The training of the considered neuro-fuzzy model consists in finding (NM+1)m of parameters $b_{jl}^k$of models (2) that provide the best image processing quality in terms of the accepted learning criterion. For this, expression (3) is rewritten in the following form

$$\begin{aligned}
\tilde{q}^k &= \sum_{j=1}^{m} \bar{w}_j^k r_j^k(q^k) = \sum_{j=1}^{m} \bar{w}_j^k \left(b_{j0}^k + \sum_{l=1}^{N\times M} b_{jl}^k q_l^k \right) = \\
&= \bar{w}_1^k (b_{10}^k + b_{11}^k q_1^k + ... + b_{1N\times M}^k q_{1N\times M}^k) + \\
&+ \bar{w}_2^k (b_{20}^k + b_{21}^k q_1^k + ... + b_{2N\times M}^k q_{2N\times M}^k) + ... + \\
&+ \bar{w}_m^k (b_{m0}^k + b_{m1}^k q_1^k + ... + b_{mN\times M}^k q_{mN\times M}^k).
\end{aligned}$$

We introduce transformation vectors

$$r^k(q^k) = (\bar{w}_1^k, \bar{w}_1^k q_1^k, ..., \bar{w}_1^k q_{1N\times M}^k, \bar{w}_2^k,\ \bar{w}_2^k q_1^k, ..., \bar{w}_2^k q_{2N\times M}^k, \bar{w}_m^k,\ \bar{w}_m^k q_1^k, ..., b_{mN\times M}^k q_{mN\times M}^k)^T$$

We introduce the vectors of estimated parameters

$$b^k = (b_{10}^k, b_{11}^k, ..., b_{1N\times M}^k, b_{20}^k, b_{21}^k ..., b_{2N\times M}^k, ..., b_{m0}^k, b_{m1}^k, ..., b_{mN\times M}^k)$$

of $(N \times M + 1)m$ dimension.

Then the generalized model can be written in a vector form

$$\tilde{q}^k = b^{kT} r^k(q^k)$$

or in a matrix form

$$\tilde{Q} = BR$$

Then the problem in a matrix form is posed as follows: find vector B so that [40, 41]:

$$E = (Q - \tilde{Q})^T (Q - \tilde{Q}) \to \min$$

where

$$\tilde{Q} = BR$$

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VI. FINDING THE PARAMETERS OF THE FUZZY MODEL

The minimum value of the quadratic residual E is reached for $\tilde{Q} = Q$, which corresponds to the solution of the equation

$$Q = BR \tag{4}$$

For real problems, the number of adjustable parameters is less than the size of the data sample, so equation (4) does not have an exact solution. In this case, the solution can be found using pseudo-inversion of matrix R:

$$B = \left(R^T R\right)^{-1} R^T Q \tag{5}$$

The problems of finding a solution to (5) are related to the possible singularity of matrix $\left(R^T R\right)$. In this regard, a fuzzy approach is proposed in the article.

1. Let b^i be triangular fuzzy number, $i = \overline{1-(N \times M+1)m}$:

$$\mu\left(b^i\right) = \begin{cases} \dfrac{r-\beta_i}{c_i} & r > \beta_i, \\ \dfrac{\beta_i - r}{c_i} & r \le \beta_i. \end{cases}$$

A fuzzy number b^i in parametric form can be written as:

$$b^i = (\beta_i,\ c_i)$$

Here β_i is the center of the interval and c_i is the width of the interval, $c_i \ge 0$.

Then the solution of problem (4) can be expressed as follows:

It is necessary to find the parameters (β_i, c_i) of fuzzy coefficients that satisfy the following conditions:

a) the values of q_k belong to the estimation interval;

b) the sum of intervals should be minimal.

These requirements can be solved using the following linear programming problem:

$$\begin{cases} \sum\limits_k \sum\limits_j c_j \left|r_{kj}\right| \to \min, \\ \sum\limits_i \beta_i r_{ki} - \sum\limits_i c_i \left|r_{ki}\right| \subset K \end{cases}$$

or

$$\begin{cases} \sum\limits_k \sum\limits_j c_j \left|r_{kj}\right| \to \min, \\ \sum\limits_i \beta_i r_{ki} - \sum\limits_i c_i \left|r_{ki}\right| \le q_k, \\ \sum\limits_i \beta_i r_{ki} + \sum\limits_i c_i \left|r_{ki}\right| \ge q_k. \end{cases} \tag{6}$$

2. Let b^i be a Gaussian fuzzy number $i = \overline{1-(N \times M+1)m}$:

$$\mu_G(r) = e^{-\frac{1}{2}\frac{(r-\beta)^2}{c^2}}$$

A fuzzy number b^i in parametric form can be written as:

$$\gamma^i = (\beta_i,\ c_i)$$

Here β_i is the center of the interval, and is the width of the interval of -level, $c_i \ge 0$.

The problem is set as follows:

Find coefficients b^i with parameters (β_i, c_i)

a) the value of $q_k$belongs to the set of estimates no lower than the α_s-level;

b) let the estimate interval of the α_s-level be minimal.

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The estimate interval of the α_s-level is found as follows:

$$d_{\alpha_s} = q_2 - q_1 = 2(q_2 - \beta)$$

Here $q_2 - \beta$ is determined from the following condition:

$$\alpha_s = \exp\left\{-\frac{1}{2}\frac{(q_2-\beta)^2}{c^2}\right\} \Rightarrow d_{\alpha_s} = 2(q_2-\beta) = 2c\sqrt{-2\ln\alpha_s}$$

For the k-th line:

$$d^k_{\alpha_s} = 2c_k\sqrt{-2\ln\alpha_s}$$

In this case, the target function can be represented as follows:

$$\min\sum_{k=1}^{m} d^k_{\alpha_s} = \min\sum_{k=1}^{m} 2c_k\sqrt{-2\ln\alpha_s} = 2\sqrt{-2\ln\alpha_s}\sum_{k=1}^{m}\sum_{i=1}^{n} c_i\,|r_{ik}|$$

According to condition a):

$$\mu(q_k) \geq \alpha_s \Rightarrow \exp\left\{-\frac{1}{2}\frac{(q_k-\beta_k)^2}{c_k^2}\right\} \geq \alpha_s \Rightarrow$$

$$\begin{cases}\beta_k + c_k\sqrt{-2\ln\alpha_s} \geq q_k,\\ \beta_k - c_k\sqrt{-2\ln\alpha_s} \leq q_k.\end{cases} \beta_k = \sum_{i=1}^{n}\beta_i r_{ki}, c_k = \sum_{i=1}^{n} c_i\,|r_{ki}|.$$

To find the fuzzy model coefficients, we solve the following linear programming problem:

$$\begin{cases}\sum\limits_k\sum\limits_j c_j\,|r_{kj}| \to \min,\\ \sum\limits_i \beta_i r_{ki} + \sqrt{-2\ln\alpha_s}\sum\limits_i c_i\,|r_{ki}| \subset K\end{cases}$$

or

$$\begin{cases}\sum\limits_k\sum\limits_j c_j\,|r_{kj}| \to \min,\\ \sum\limits_i \beta_i r_{ki} + \sqrt{-2\ln\alpha_s}\sum\limits_i c_i\,|r_{ki}| \geq q_k,\\ \sum\limits_i \beta_i r_{ki} - \sqrt{-2\ln\alpha_s}\sum\limits_i c_i\,|r_{ki}| \leq q_k.\end{cases} \tag{7}$$

The parametric model S with independent parameters $\alpha_1, \alpha_2, ..., \alpha_S$ (7) or parametric problem S in a matrix form is written as follows:

$$z = rc \to \min$$

$$r(\beta + \alpha c) \subset K \tag{8}$$

$$\alpha \in R^s$$

Here $K = \left\{ q|\ \ q \in R^l, q \leq |q_k| \right\}$ is a given convex subset of space R^l.

A problem of this type can be called a parametric programming problem with multi-valued coefficients. It is clear that within the framework of this problem it makes no sense to talk about maximizing the target function, since the

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values of this function are not numbers, but a set of numbers. In this case, it is necessary to find out what preference relation in the set of alternatives this function generates, and then to investigate the question of what choices are considered rational in the sense of this preference relation.

The next step towards refining the considered model is fuzzification, i.e. descrip-tion of the coefficients of the problem in the form of fuzzy sets. In this case, in addition to setting the sets of possible parameter values, additional information is introduced into the model in the form of membership functions of these fuzzy sets. Thus, we have come to the formulation of the problem of fuzzy parametric programming. Problem (9) is reduced to the following problem of parametric programming

$$z = rc \to \min,$$

$$r(\beta + \alpha c) \le q, \tag{9}$$

$$\alpha \in R^S.$$

To find alternatives the degree of non-dominance of which is no less than , it is enough to solve the following problem of mathematical programming for the case un-der consideration:

$$z(\alpha) \to \min,$$

$$\varphi(r, z(\alpha)) \ge \alpha,$$

$$r(\beta + \alpha c) \le q,$$

$$\psi(\alpha) = r(\beta + \alpha c) - q \le 0,$$

where φ is the membership function $z(\alpha)$.

A solution to problem (9) is an explicitly given decision function

$$z_s(\alpha) = \{z = rc \mid r(\beta + \alpha c) \le q; \beta \ge 0,\ c > 0\}$$

We denote the set of solutions α satisfying (9) by O_i. Due to the fact that the values of the coefficients are fuzzy, O_i is a fuzzy set with a membership function $\varphi_{0i}(\alpha)$. In this way,

$$O_i = \{\alpha, \varphi_{0i}(\alpha), \alpha \in R^S\}$$

The fuzzy solution corresponds to the fuzzy maximum value

$$\varphi_{zi} = \sup_{\alpha \in z^{-1}} \varphi_{0i}(t)$$

For any allowable vector β, c, the following inequalities are true

$$z_s(\alpha) \le g(\alpha) where g(\alpha) = rc. \tag{10}$$

The mapping q given by (9) is described by the following expression

$$\varphi_q(z_S(\alpha), g(\alpha)) = \begin{cases} 1, & g(\alpha) \ge z_s(\alpha), \\ 0, & g(\alpha) < z_s(\alpha). \end{cases} \tag{11}$$

Due to the fact that $z_s(\alpha)$ is an element of the fuzzy set Z_α, g(t) will belong to some fuzzy set G_α, which is the image of Z_α under the mapping q. According to [38], the membership function of set $Z_\alpha$and its image $G_\alpha$are related by the following relationship

$$\varphi_{G_\alpha}(g(\alpha)) = \max_{z_s(\alpha)} \min\{\varphi_{Z_\alpha}(z_s(\alpha)), \varphi_q(z_S(\alpha), g(\alpha))\} \tag{12}$$

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Substituting $\varphi_q(z_s(\alpha), g(\alpha))$from (8) into (11), we

$$\varphi_{G_\alpha}(g(\alpha)) = \max_{z_s(\alpha)} \{\alpha_{Z_\alpha}(z_s(\alpha)) : z_S(\alpha) \leq g(\alpha)\} \tag{13}$$

For the function $g(\alpha)$, the set O_i is a preimage of G_α. Hence, according to [64], we have

$$\phi_{0i}(t) = \phi_{G_t}(g(t)) = \phi_{G_t}((a_0^{'} + t^{'}b) + e^{'}t) \tag{14}$$

Substituting (12) into (13), we obtain

$$\varphi_{0i}(\alpha) = \max_{z_s(\alpha)} \{\varphi_{Z_\alpha}(z_s(\alpha)) : z_s(\alpha) \leq rc\} \tag{15}$$

Solution (15) for $\varphi_{Z_\alpha}(z_s(\alpha))$ has the following form

$$\varphi_{z_i}(\alpha) = \exp(\frac{k_i}{2}[\max(0, z_s(\alpha) - rc)]^2) \tag{16}$$

The fuzzy mapping depends on s variables, i.e. $\alpha = \alpha_1 \times \alpha_2 \times ... \times \alpha_S$ is the Cartesian product of the corre-sponding sets. In the general case, the membership function of this subset has the form

$$\varphi_z(\alpha) = \prod_{i=1}^{s} \varphi_{z_i}(\alpha) \tag{17}$$

From (16) and (17) we have

$$\varphi_z(\alpha) = \exp(-\Phi(\alpha)), \tag{18}$$

where

$$\Phi(\alpha) = \frac{1}{2}\sum_{i=1}^{s} k_i[\max(0, z_s(\alpha) - rc)]^2.$$

We have proved the following assertion 1: Let $z_s(\alpha)$ be a fuzzy solution of problem (8). Then the set of solutions has the membership function (17).

If there is fuzzy information, then one needs to choose such an estimate of param-eter α, so that it minimizes $z_s(\alpha)$ and at the same time its membership degree to the allowable set $Z_\alpha$is maximal, i.e. :

$$z(\alpha) \to \min, \varphi_z(\alpha) \to \max \tag{19}$$

According to Lemma 2 [23, p.61], due to the positiveness of $\varphi_z(\alpha)$, the second criterion in (18) can be replaced by $Ln\varphi_z(\alpha)$. Thus, (18) is equivalent to the problem

$$z(\alpha) \to \min, \Phi(\alpha) \to \min \tag{20}$$

The preferred solutions to such a problem are solutions that cannot be improved by one criterion without increasing the other criterion in (19). Function $z(\alpha)$ is a convex function, $\Phi(\alpha)$is a convex func-tion. In this case, Pareto optimal solutions are the solution to the problem [23]

$$\begin{aligned} &L(\alpha) = z(\alpha) + h\Phi(\alpha) \to \min \\ &h \in [0, \infty] \end{aligned} \tag{21}$$

In the case of formalization of information in the form of fuzzy sets, not one solu-tion is determined, but a certain set of them. They are function of h. Let us denote them by $\alpha(h)$.

Due to the convexity of function $L(\alpha)$for $h_2 > h_1 \geq 0$ we have

$$z(h_1) + h_1\Phi(h_1) < z(h_2) + h_1\Phi(h_2),$$

$$z(h_2) + h_2\Phi(h_2) < z(h_1) + h_2\Phi(h_1).$$

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From here we get

$$h_2(\Phi(h_2) - \Phi(h_1)) + h_1(\Phi(h_1) - \Phi(h_2)) < 0,$$

$$(\Phi(h_2) - \Phi(h_1))(h_2 - h_1) < 0.$$

Assertion 2 follows from this inequality: If $L(\alpha)$is a convex function, then $\varphi_z(\alpha)$- strictly monotonically decreases at $h \geq 0$.

This means that there is no alternative in the set $O_i$for which the fol-lowing inequalities would hold simultaneously

$$\varphi_o(\alpha) > \varphi_z(h) > 0 \text{ and } z(\alpha) > h,$$

i.e., there is no such element α that would have a greater than $\varphi_z(h)$, membership degree to the set φ_0 and would give more than h of the maximized function. If the decision maker prefers to choose a specific alternative α as a decision, then his choice should be based not only on the membership degree of this alternative to the fuzzy set $\varphi_z(\alpha)$, but also on the corresponding values of function $z(\alpha)$.

Then the fuzzy number parameters q are in the following form:

center of the interval:

$$b_q = \sum_j \beta_j r_{ij} = \beta^T r,$$

width of the interval

$$c_q = \sum_j c_j \left| r_{ij} \right| = c^T \left| r \right|.$$

VII. COMPUTATIONAL EXPERIMENT

The base of rules for quantifying these parameters for the original image and the image obtained after processing is given in Table 1. When compiling the logical rules, it was taken into account that the image quality deteriorates at the presence of noise, poor brightness and contrast. When these factors are absent or reduced, the image quality improves. The image quality estimation model is based on Sugeno's fuzzy in-ference, in which all values of input and output variables are given by fuzzy sets.

The input signal is given as a matrix, the columns of which correspond to different classes of the processed image. As mentioned above, in the zero layer, this signal is converted into a vector with elements $q_i^k$where$i = \overline{1, N \times M}$, k takes values from zero to 3 in our problem, i.e. $q_i^0 = f$,$q_i^1 = g_1$, $q_i^2 = g_2$, $q_i^3 = g_3$, respectively.

Table 1. Rule base of quantitative estimate.

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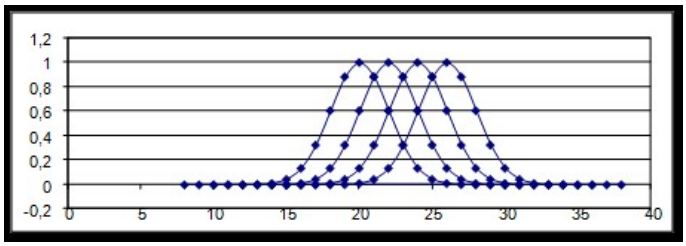


FIG. 6. Membership function of output terms before training.

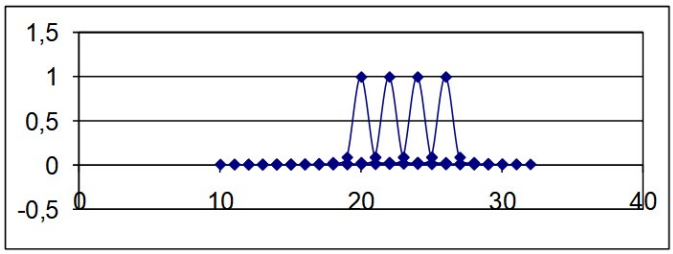


FIG. 7. Membership functions of input terms after training.

Rules	**IF inputs**				**"THEN" inputs Image quality r**
	Brightness q_1	**Contrast** q_2	**Noise** q_3	**Shift in focus** q_4	
1	B	B	B	B	r=1,6+4,7q_1-66,9q_2+37,4q_3+2,8q_4
2	B	B	B	G	r=1,6+30,4q_1-71,7x_2+52,1q_3-0,7q_4
3	B	B	B	E	r=2,0+392,5q_1-354,5q_2+12,8q_3-4,7q_4
4	B	B	G	B	r=0,3+0,1q_1+50,1q_2+3,0q_3-4,7q_4
5	B	B	G	G	r=-3,2-13,5q_1+35,4q_2+9,9q_3-2,0q_4
6	B	B	G	E	r=1,2-3,1q_1+1,1q_3-0,6q_4
7	B	B	E	B	r=2,1+28,7q_1+56,1q_2-0,1q_3-8,7q_4
8	B	B	E	G	r=-0,7+64,0q_1+31,1q_2+2,7q_3-9,5q_4
9	B	B	E	E	r=5,5+6,4q_1-10,6q_2-3,4q_3+0,7q_4
10	B	G	B	B	r=2,7-64,4q_1-1,9q_2-6,6q_3+7,9q_4
11	B	G	B	G	r=0,8+28,9q_1+2,0q_2-10,2q_3-1,2q_4
12	B	G	B	E	r=5,5-3,8q_1-7,4q_2+2,0q_3+1,1q_4
13	B	G	G	B	r=-1,5+158,3q_1+5,5q_2+1,6q_3-16,2q_4
14	B	G	G	G	r=-2,5-99,7q_1+8,8q_2+0,2q_3+9,4q_4
15	B	G	G	E	r=-6,3-7q_1-1,2q_2+17,9q_3-1,6q_4
16	B	G	E	B	r=1,6+124,0q_1-0,3q_2+0,7q_3-12,0q_4
17	B	G	E	G	r=3,5+31,7q_1-1,9q_2-0,4q_3-2,8q_4

The membership functions of the fuzzy terms used in this knowledge base were chosen expertly (Figure 6). The neuro-fuzzy network selected by these membership functions provides a rough approximation of the object.

The membership functions of fuzzy terms after training are shown in Figure 7.

From figures 6, 7 and table 2, it can be seen that as a result of training the neuro-fuzzy network, the parameters with the membership function have undergone the greatest changes

$$\mu_G(r) = e^{-\frac{1}{2}\frac{(r-\beta)^2}{c^2}},$$

while other parameters remained virtually unchanged. The expert's choice of large values of parameter indicates a

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significant uncertainty in the estimation of fuzzy terms. The decrease in the values of parameters c during training led to a "con-centration" of membership functions, which indicates the removal of uncertainty in the estimates of fuzzy terms.

$\frac{\partial\mu}{\partial A}$	$\frac{\partial\mu}{\partial\beta}$	$\frac{\partial E}{\partial c}$	$\frac{\partial E}{\partial\beta}$	c	β
Before training					
				1	0,32
				1	0,30
				1	0,29
				1	0,27
				1	0,26
				1	0,25
				1	0,23
				1	0,22
				1	0,21
				1	0,19
After training					
0,21	1,20	-8,12	-41,86	0,29	0,30
0,11	0,93	-4,6	-32,47	0,29	0,28
0,04	0,60	-1,97	-10,28	0,29	0,26
0,22	0,28	3,27	4,10	0,28	0,25
0	0	0	0	0,29	0,23
0,02	-0,28	0,17	-4,10	0,28	0,22
0,03	-0,50	0,68	-8,86	0,28	0,21
0,09	-0,81	1,79	-5,10	0,28	0,20
0,17	-1,07	3,17	-5,10	0,28	0,18
0,27	-1,30	4,84	-5,09	0,28	0,24

Fuzzy logic methods are a powerful tool for representing and processing knowledge in the form of "IF-THEN" rules, fuzzy inference systems are developed faster and are simpler than their sharp analogs.

VIII. CONCLUSIONS

An analysis of literature sources on the use of fuzzy methods in digital image processing shows that these methods are mainly used in the following tasks: image contrast enhancement, edge detection and segmentation. However, these methods are not widely used in solving applied problems of image processing due to their computa-tional complexity. Taking this into account, the purpose of the study defined in the ar-ticle is image processing using the apparatus of fuzzy sets. The composition and struc-ture of the image processing system based on the theory of fuzzy sets were determined. A block diagram of the main functional subsystems was developed, taking into ac-count the use of "soft computing". A scheme for preliminary processing and assessment of image quality using the apparatus of fuzzy sets was proposed. The issues of forming rule bases were worked out on a model example. By means of a computer experiment, it was shown that the use of the proposed neuro-fuzzy network significantly reduces the cost of training. For a neural network, the number of training iterations is 8400, and the time is 7 minutes; for a neuro-fuzzy network, the number of iterations is 1150, and the time is 1.2 min.

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