Journal of Fundamental and Applied Research **Vol. 2**, Issue 3 (2022) 20220020

Received: 27 July 2022 / Accepted: 06 September 2022
Published online: 25 September 2022
© 2022 The Author(s)

Five dimensional Reissner-Nordström black hole can be overcharged

Sanjar Shaymatov,[1,2,3,4,*] Odiljon Sultonov,[2,†] and Farid Sattorov[5]

[1] *Institute of Fundamental and Applied Research, National Research University TIIAME, Kori Niyoziy 39, Tashkent 100000, Uzbekistan*
[2] *National University of Uzbekistan, Tashkent 100174, Uzbekistan*
[3] *Tashkent State Technical University, Tashkent 100095, Uzbekistan*
[4] *Samarkand State University, University Avenue 15, Samarkand 140104, Uzbekistan*
[5] *The National Institute of Fine Arts and Design named after Kamoliddin Behzod, Mironshox 123, Tashkent 100000, Uzbekistan*

We investigate, under specific conditions, the effect of a charged test particle on the process of overcharging a five dimensional charged black hole in Einstein gravity. We show that it is possible to violate the weak cosmic censorship conjecture for a five dimensional charged black hole by throwing in a charged particle with suitable parameters, similarly to what is observed in the Reissner-Nordström black hole in four dimension.

PACS numbers: 04.70.Bw, 04.20.Dw

I. INTRODUCTION

Since cosmic censorship hypothesis [1] has remained unproven yet, the physical possibility of destroying the event horizon of black holes with test particles or fields have recently become the object of active scientific research. However, in general relativity, a naked singularity is still an unanswered question. Black holes start taking center stage by the LIGO-VIRGO detection of stellar black hole mergers [2, 3]. The hidden properties of black holes could be probed by analyzing gravitational waves in the near future.

An experiment in destroying a rotating black hole was demonstrated by Wald [4] for the first time, according to which it is impossible to destroy an extremal Kerr-Newman black hole by infalling test particles. However, a real interest in such a destroying problem has been recently grown, particularly, after Jacobsan and Sotirio (JS) [5] shown that it would be possible to overspin the rotating black hole by means of infalling non-spinning test particles if only one starts a near extremal one in the sense of neglecting the backreaction effects. It is worth noting that Hubeny [6] argued the possibility of destroying a Reissner-Nordström black hole. Recently, it was also shown by Saa and Santarelli [7] that it would be possible to destroy a Kerr-Newman black hole with a charged particle. In view of the current thought experiment, turning black holes into naked singularity with the help of infalling test particles has been investigated as a very active field of research. An extensive discussion of the possibility of destroying the event horizon by imposing the condition of nearly extremal black hole in various background geometry can be found in [see, e.g. 5–10]. Recently, the mechanism of destroying a Kerr-Newman-AdS black hole has been constructed by neglecting radiative and self-force effects [11–13]. In all these works, it was assumed that the test particle follows a geodesic motion and backreaction effects were not considered. In the fact that the impossibility of destroying black hole horizons relies on radiation reaction and self-force effects for which the infalling particles could be unable to enter a black hole [14–16]. It was also shown in our previous work [10] that it would be possible to destroy near-extremal Kerr black hole with a charged particle and a test magnetic field would act as a cosmic censor beyond a certain value of the magnetic field. In case when the backreaction and higher order perturbations are taken into account,

*sanjar@astrin.uz
†sultonovodiljon2989@gmail.com

the possibility of destroying black holes in different gravity models has been investigated in Refs. [17–24]. Also, there are investigations that tested the validity of cosmic censorship in the rotating anti-de Sitter black holes [25, 26]. This conjecture has been also analyzed in Ref. [27] as stated by the laws of black hole dynamics [28]. It is worth noting that the gedanken experiment [4] was also extended to a magnetized black hole case [29, 30]. The test of weak cosmic censorship conjecture was also considered in the case of BTZ black holes [31, 32].

It is of particularly importance understanding more deeply the properties of higher dimensional black holes by studying the process of overcharging a black hole by plugging in test particles. With this motivation, in this paper, we test the weak cosmic censorship conjecture for five dimensional charged black hole in Einstein gravity.

In Sec. II we briefly describe the five dimensional charged black hole metric, the dynamics of test particle motion and the process of overcharging a nearly extremal black hole by throwing in a charged particle. We summarize our concluding remarks of the obtained results in the Sec. III. Throughout the manuscript we use a system of units in which $G = c = 1$.

II. PARTICLE MOTION AND THE WEAK COSMIC CENSORSHIP CONJECTURE FOR A NEAR-EXTREMAL FIVE DIMENSIONAL REISSNER-NORDSTRÖM BLACK HOLE

In this paper, we study the weak cosmic censorship conjecture in five dimensional charged black hole in Einstein gravity using test particles. We then try to understand whether it is possible to overcharge a black hole and then turn it into a naked singularity. For that we first consider a charged particle motion around a Reissner-Nordström black hole by applying appropriate values of parameters that is able to overcharge a near-extremal Reissner-Nordström black hole in four dimension [6]. The main motivation here is to make sure that whether or not the charged particle with given suitable parameters can be prevented from plunging into black hole, thereby not violating the weak cosmic censorship.

The metric, which describes five dimensional spherically symmetric charged black hole, is given by

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\theta^2 + r^2 d\Omega^2_{D-2}\,, \tag{1}$$

where Ω^2_{D-2} is the line element of D sphere and

$$f = 1 - \frac{2M}{r^{D-3}} + \frac{Q^2}{r^{2(D-3)}}\,. \tag{2}$$

Here, Q is the electric charge of the black hole.

The corresponding electromagnetic potential has the form as

$$A = -\frac{Q}{r^{D-3}} dt\,. \tag{3}$$

We now attempt to find the horizon radius of the Reissner-Nordström black hole in five dimension. For horizon radius the condition $Det[g_{\alpha\beta}] = 0$ must be satisfied and gives the following equation

$$r^2(D-3) - 2Mr^{D-3} + Q^2 = 0\,, \tag{4}$$

for which the radius of horizons has the form

$$r_\pm = \left[M + \sqrt{M^2 - Q^2}\right]^{\frac{1}{D-3}}\,. \tag{5}$$

From Eq. (5), the horizon stability always holds well provided that $M > Q$ is satisfied only, whereas $M < Q$ leads horizon to be destroyed. To reach the latter condition we consider a charged particle motion falling radially toward the five dimensional Reissner-Nordström black hole with mass m and charge q. We assume conserved energy $\delta M \ll M$ for the test particle approximation. The falling of the charged particle into the black hole transfers the mass and charge to its mass and charge. Hence, the final state of the black hole parameters will have mass $M + \delta M$ and total charge $Q + q$, respectively.

Now we deal with the charged particle that is allowed to enter the black hole with suitable parameters. Thus, we understand the behavior of the radial motion of the particle whether or not the particle crosses the horizon. Its radial motion can be demonstrated in terms of the effective potential. Following the equation of particle motion in five dimensional black hole using Eq. (3), the effective potential for particle motion takes the form as

$$V_{eff}(r) = \frac{1}{2} - \frac{M}{r^{D-3}} + \frac{Q^2}{2r^{2(D-3)}} - \frac{\left(\delta M + qA_t(r)\right)^2}{2m^2}\,. \tag{6}$$

Copyright © 2022

For a charged particle to enter the black hole it never feels any restriction, for which the effective potential must be always negative outside the horizon. With this motivation, we should find reasonable test particle parameters allowing to cross the horizon without any restriction. Thus, we first assume that the charged test particle falls toward the black hole either along the axis or radially with zero angular momentum $\delta J = 0$. As long as a charged particle absorbs by the black hole, its absorption adds to the mass and charge of the black hole. As a result, the final parameters of the black hole are given by $\bar{M} + \delta M$ and $\bar{Q} + q$, respectively. Thereafter, the black hole can be overcharged when the condition below is satisfied only:

$$(\bar{M} + \delta M)^2 < (\bar{Q} + q)^2 \,, \tag{7}$$

for the lower and an upper bounds on the energy of the charged particle

$$\frac{qQ}{r_+^2} = \delta M_{min} < \delta E \,, \tag{8}$$

$$\delta M < \delta M_{max} = \bar{Q} + q - \bar{M} \,. \tag{9}$$

Note that the black hole must start out very close to extremal. Conditions above (8) and (9) can not be satisfied simultaneously in the extremal case $Q = M = r_+$, thereby being broken. Taking into account conditions above we study the effective potential for radial motion of charged particle falling in toward the black hole. The effective potential can be described in terms of the energy, charge, and mass of the charged particle

$$V_{eff}(r) \;=\; \frac{M}{r^2}\left(\frac{Q}{M}\frac{\delta M q}{m^2} - 1\right) - \frac{1}{2}\left(\frac{\delta M^2}{m^2} - 1\right) - \frac{Q^2}{2r^4}\left(\frac{q^2}{m^2} - 1\right) . \tag{10}$$

Now we investigate the effective potential. The effective potential has turning points at which $V_{eff}(r) = 0$. However, this effective potential also has maximum at

$$r_{max} = \left[Q\left(\frac{q^2}{m^2} - 1\right)\left(\frac{\delta M q}{m^2} - \frac{M}{Q}\right)^{-1}\right]^{1/2} . \tag{11}$$

For this maximum we must choose reasonable parameters for particle for which the effective potential is negative, so that particle starts crossing through the horizon swimmingly. For this purpose we take those parameters used in [6] so as to test whether or not the particle can fall across the horizon. Thus, we focus on those appropriate parameters for exploring them numerically:

$$\begin{aligned} Q &= 1 - 2\epsilon^2 \\ q &= \alpha\epsilon \quad \text{with} \quad \alpha > 1 \\ \delta M &= \alpha\epsilon - 2\beta\epsilon^2 \quad \text{with} \quad 1 < \beta < \alpha \\ m &= \gamma\epsilon \quad \text{with} \quad \gamma < \sqrt{\alpha^2 - \beta^2} \,, \end{aligned} \tag{12}$$

and they can easily satisfy conditions (8) and (9) with a suitable choice of parameters α, β, and γ. So for the given background spacetime with parameters $M = 1$ and $Q = 0.99999$ as stated in [6] we suppose the particle has charge $q = 3 \times 10^{-3}$. In this sense the particle, thrown radially inward the black hole, has the allowed range of energy for the chosen charge parameter q

$$2.9866 \times 10^{-3} < \delta M < 2.9900 \times 10^{-3} \,, \tag{13}$$

which lets to obtain a suitable range of mass m. Using choice of parameters, we finally find the appropriate values of the mass

$$m \lesssim 1.96 \times 10^{-3} \,, \tag{14}$$

Throwing in a charged particle inward the black hole radially or along the axis with $\delta J = 0$, We must then be sure that whether the particle with suitable parameters enters the black hole, thus converting near extremality to over extremality. For charged particles to enter the black hole without any restriction Eq. (10) must be negative or $\dot{r}^2$ must be positive everywhere outside the horizon. For this, we address the radial motion of charged test particle given simply in terms of the effective potential $\dot{r}^2 = -2V_{\rm eff}(r, \delta\tilde{M}, q)$. In Table I, we have tabulated the maximum values of the effective potential for the various combinations of particle parameters. For those allowed values, we examine the behavior of the effective potential numerically. As can be seen from the Table I, for each allowed value the effective potential is negative, allowing test particle to fall across the horizon. Thus, as can be seen from Table I, it would be possible for the charged particle with suitable parameters to fall across the horizon, thus violating the weak cosmic censorship conjecture for the five dimensional Reissner-Nordström black hole and happening so-called "jump" from near extremality to over extremality.

Copyright © 2022

TABLE I: The values of r_{max} outside the horizon and effective potential is tabulated for the different values of energy and mass along with the critical value of the magnetic field. The charge here is considered as $q = 3 \times 10^{-3}$.

m	δM	r_{max}	V_{eff}
0.9×10^{-3}	2.9874×10^{-3}	1.004630	-4.2×10^{-6}
1.4×10^{-3}	2.9884×10^{-3}	1.004960	-2.17×10^{-6}
1.8×10^{-3}	2.9897×10^{-3}	1.005390	-1.18×10^{-6}
1.85×10^{-3}	2.9899×10^{-3}	1.005460	-1.07×10^{-6}

III. CONCLUSION

In this paper, we studied the process of overcharging of five dimensional charged black hole via the charged particle absorption whether the weak cosmic censorship conjecture is violated or not. The general form of effective potential has been obtained for the radially falling charged particle. We also determined the allowed range of of energy for the charged particle for given charge parameter for five dimensional charged black hole. Taking altogether we showed that five dimensional charged black hole in Einstein gravity can be overcharged so that it is taken to a new black hole state with final parameters $(M + \delta M, Q + q)$ by test particle absorption. Hence, it is possible to violate the weak cosmic censorship conjecture for the charged black hole in five dimension, thus occurring so-called "jump" from near extremality to over extremality and turning into the final object that is parametrically and physically different with the initial black hole.

[1] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969).
[2] B. P. Abbott and et al. (Virgo and LIGO Scientific Collaborations), Phys. Rev. Lett. **116**, 061102 (2016), arXiv:1602.03837 [gr-qc] .
[3] B. P. Abbott and et al. (Virgo and LIGO Scientific Collaborations), Phys. Rev. Lett. **116**, 241102 (2016), arXiv:1602.03840 [gr-qc] .
[4] R. Wald, Ann. Phys. (N.Y.) **82**, 548 (1974).
[5] T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. **104**, 021101 (2010), arXiv:0911.3363 [gr-qc] .
[6] V. E. Hubeny, Phys. Rev. D **59**, 064013 (1999), gr-qc/9808043 .
[7] A. Saa and R. Santarelli, Phys. Rev. D **84**, 027501 (2011), arXiv:1105.3950 [gr-qc] .
[8] G. E. A. Matsas and A. R. R. da Silva, Phys. Rev. Lett. **99**, 181301 (2007), arXiv:0706.3198 [gr-qc] .
[9] E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius, and U. Sperhake, Phys. Rev. Lett. **103**, 239001 (2009), arXiv:0911.2243 [gr-qc] .
[10] S. Shaymatov, M. Patil, B. Ahmedov, and P. S. Joshi, Phys. Rev. D **91**, 064025 (2015), arXiv:1409.3018 [gr-qc] .
[11] K. Düztaş, Class. Quantum Grav. **35**, 045008 (2018), arXiv:1710.06610 [gr-qc] .
[12] B. Gwak, J. High Energy Phys. **09**, 81 (2018), arXiv:1807.10630 [gr-qc] .
[13] K. Düztaş and M. Jamil, Mod. Phys. Lett. A **34**, 1950248 (2019), arXiv:1812.06966 [gr-qc] .
[14] E. Barausse, V. Cardoso, and G. Khanna, Phys. Rev. Lett. **105**, 261102 (2010), arXiv:1008.5159 [gr-qc] .
[15] P. Zimmerman, I. Vega, E. Poisson, and R. Haas, Phys. Rev. D **87**, 041501 (2013), arXiv:1211.3889 [gr-qc] .
[16] J. V. Rocha and V. Cardoso, Phys. Rev. D **83**, 104037 (2011), arXiv:1102.4352 [gr-qc] .
[17] Z. Li and C. Bambi, Phys. Rev. D **87**, 124022 (2013), arXiv:1304.6592 [gr-qc] .
[18] S. Shaymatov, N. Dadhich, and B. Ahmedov, Eur. Phys. J. C **79**, 585 (2019), arXiv:1809.10457 [gr-qc] .
[19] J. Jiang and M. Zhang, Eur. Phys. J. C **80**, 196 (2020).
[20] S. Shaymatov and N. Dadhich, JCAP **2022**, 060 (2022), arXiv:2008.04092 [gr-qc] .
[21] S. Shaymatov, N. Dadhich, B. Ahmedov, and M. Jamil, Eur. Phys. J. C **80**, 481 (2020), arXiv:1908.01195 [gr-qc] .
[22] S. Shaymatov, N. Dadhich, and B. Ahmedov, Phys. Rev. D **101**, 044028 (2020), arXiv:1908.07799 [gr-qc] .
[23] S. Shaymatov and N. Dadhich, Phys. Dark Universe **31**, 100758 (2021), arXiv:2004.09242 [gr-qc] .
[24] S. Shaymatov, B. Ahmedov, and M. Jamil, Eur. Phys. J. C **81**, 588 (2021).
[25] B. Gwak and B.-H. Lee, JCAP **02**, 015 (2016), arXiv:1509.06691 [gr-qc] .
[26] J. Natário, L. Queimada, and R. Vicente, Class. Quantum Grav. **33**, 175002 (2016), arXiv:1601.06809 [gr-qc] .
[27] N. Dadhich and K. Narayan, Phys. Lett. A **231**, 335 (1997), arXiv:gr-qc/9704070 [gr-qc] .
[28] B. C. J.M. Bardeen and S. Hawking, Commun. Math. Phys. **31**, 161 (1973).
[29] H. M. Siahaan, Phys. Rev. D **93**, 064028 (2016), arXiv:1512.01654 [gr-qc] .
[30] S. Shaymatov, Int. J. Mod. Phys. Conf. Ser. **49**, 1960020 (2019).
[31] K. Düztaş, Phys. Rev. D **94**, 124031 (2016), arXiv:1701.07241 [gr-qc] .
[32] K. Düztaş, M. Jamil, S. Shaymatov and B. Ahmedov, Class. Quantum Grav. **37**, 175005 (2020), .

Copyright © 2022