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Particle motion around Bardeen solution with a cloud of strings

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This work is devoted to the investigation of Bardeen solution with a cloud of strings with the use of particle motion. I consider neutral particle to probe the spacetime of black hole defined by the mentioned solution. I calculate effective potential of the test particle, specific energy and specific angular momentum by solving Hamilton-Jacobi equation of motion. I show how the spacetime parameter which I call as string parameter affects these quantities.

I. INTRODUCTION

The spacetime metric of Bardeen solution with a cloud of strings was recently obtained in[1] and has the following line element

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \,, \tag{1}$$

$$f(r) = 1 - \frac{2Mr^2}{(q^2+r^2)^{3/2}} - a - \frac{\lambda r^2}{3} \,. \tag{2}$$

Here spacetime parameters M, q, λ, and a are related to black hole (BH) mass, magnetic charge, cosmological constant, and string parameter, respectively. It is worth noting that in the case $a = 0$ it recovers Bardeen-AdS solution, $a = 0$ and $\lambda = 0$ Bardeen solution, while $a = 0$, $q = 0$, and $\lambda = 0$ Schwarzschild solution. In [1] it has been shown that the string parameter can take values in the range $0 < a < 1$.

I use Hamilton-Jacobi (HJ) equation of motion to investigate neutral test particle motion around Bardeen BH with a cloud of strings. The equation reads

$$g^{\mu\nu}\frac{\partial S}{x^\mu}\frac{\partial S}{x^\nu} = -m^2 \,, \tag{3}$$

where $g^{\mu\nu}$ is contravariant components of the metric tensor, S is an action, and m defines the mass of test particle. Since the given spacetime is spherically symmetric one can write the action in the form

$$S = -Et + L\phi + R(r) + T(\theta) \tag{4}$$

with E and L are energy and angular momentum of the test particle, respectively. One can solve the equation of motion (3) taking account the above expression which results the effective potential of the test particle to take the form (when $\theta = \pi/2$)

$$V_{eff} = -\frac{\mathcal{E}^2}{a + r^2\left(\frac{\lambda}{3} + \frac{2M}{(q^2+r^2)^{3/2}}\right) - 1} - \frac{\mathcal{L}^2}{r^2} - 1 \,, \tag{5}$$

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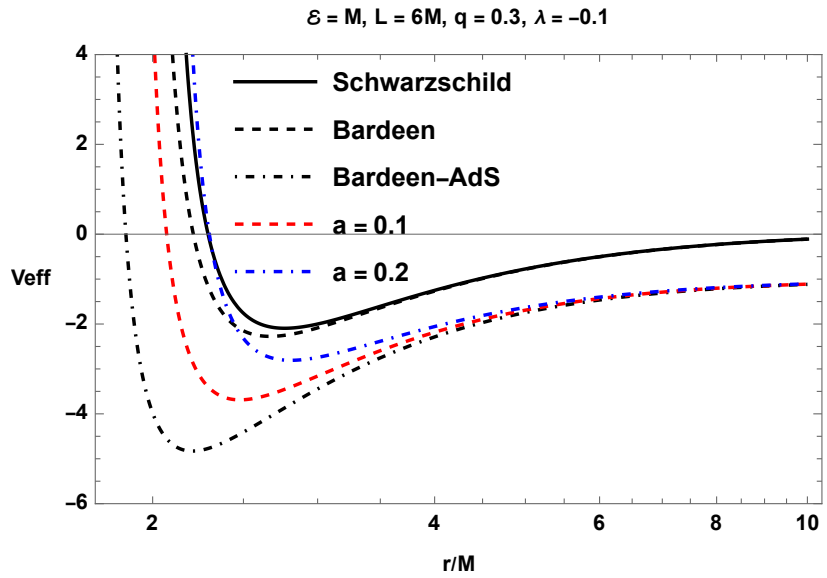


FIG. 1: Radial dependence of the effective potential of test particle orbiting Bardeen BH with a cloud of strings.

with the specific energy $\mathcal{E} = E/m$ and specific angular momentum $\mathcal{L} = L/m$. the radial dependence of the effective potential is given in Fig. 1 in comparison with the Schwarzschild, Bardeen, and Bardeen-AdS solutions. One can see that, increase of the string parameter a increases the effective potential while the presence of the cosmological constant λ decreases it.

Let us now study the case of circular orbits. This requires the effective potential to satisfy the conditions

$$V_{eff} = 0\,, \tag{6}$$
$$\partial_r V_{eff} = 0\,. \tag{7}$$

From these conditions one can find the specific energy and specific angular momentum of test particle moving on circular orbits which have the following form

$$\mathcal{E} = \frac{-a - \frac{2Mr^2}{(q^2+r^2)^{3/2}} - \frac{\lambda r^2}{3} + 1}{\sqrt{-a - \frac{3Mr^4}{(q^2+r^2)^{5/2}} + 1}} \tag{8}$$

$$\mathcal{L} = \frac{r\sqrt{r^2\left(\frac{3M(r^2-2q^2)}{(q^2+r^2)^{5/2}} - \lambda\right)}}{\sqrt{3}\sqrt{-\frac{(a-1)q^6+3(a-1)q^4r^2+3(a-1)q^2r^4+(a-1)r^6+3Mr^4\sqrt{q^2+r^2}}{(q^2+r^2)^3}}} \tag{9}$$

The dependence of the specific energy of the test particle from the radius of circular orbits is presented in Fig. 2. It is clearly seen from the plot that increase of the string parameter increases the energy needed for the test particle to move in the circular orbit of proper radius.

The radial dependence of the specific angular momentum can also be easily plotted as demonstrated in Fig. 3. One can see that specific angular momentum of the test particle behaves similarly to the specific energy of the test particle.

Conclusion

In this work I have investigate the effect of the string parameter on the motion of test particle moving in the equatorial plane of Bardeen BH with a cloud of strings. I have used Hamilton-Jacobi equation of motion to derive the effective potential of the test particle. It has been obtained explicit form of the effective potential and has been demonstrated the radial dependence graphically. It has been shown that the effective potential increases with the increase of the string parameter.

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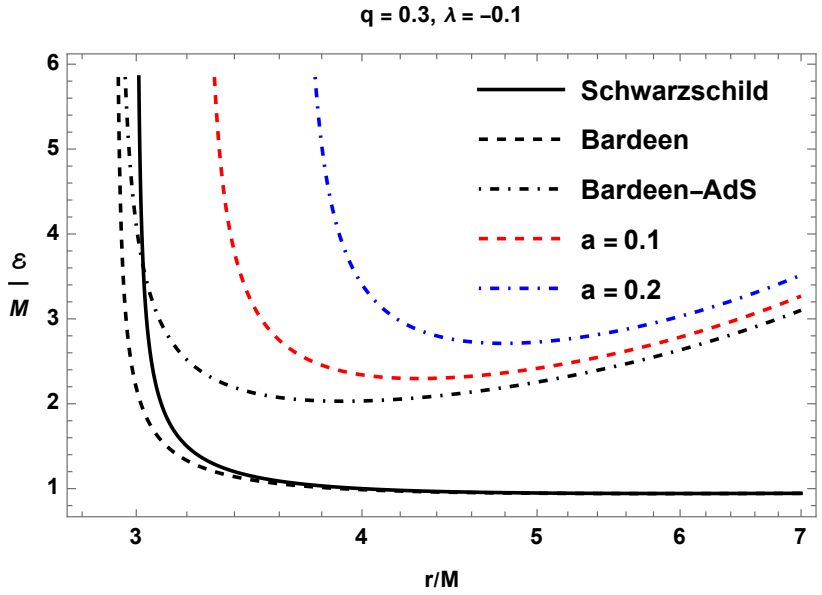


FIG. 2: Change of the Kretschmann scalar with the change of radial coordinate for different values of the spacetime parameters.

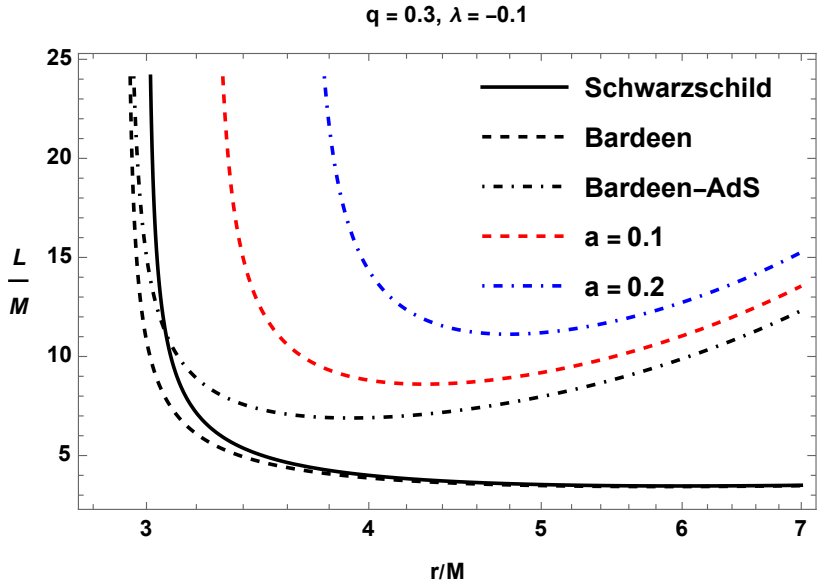


FIG. 3: Change of the Kretschmann scalar with the change of radial coordinate for different values of the spacetime parameters.

Next, I studied circular motion of test particle moving in the equatorial plane. It has been obtained explicit form of the specific energy and specific angular momentum of test particle. It has been also shown the dependence of these quantities from the radius of circular orbits. It has been observed that increase of the string parameter increases the values of these two quantities as well.

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