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Test particles around charged black holes in Born-Infeld gravity

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In this work we study circular motion of test particles around charged black holes in Born-Infeld gravity. First, we investigate radial dependence of the black hole charge. Then, we consider scalar invariants of the spacetime around Born-Infeld black holes such as Ricci scalar, square of Ricci tensor and Kretchmann scalar. We alayse the effects of the black hole charge and the Born-Infeld parameter on the energy and angular momentum of the particles corresponding to circular motion of the particles. Finally, we also investigate behaviour of the radius of innermost stable circular orbits of test particles with respect to the variation of the black hole charge for the different values of the Born-Infeld parameter.

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I. INTRODUCTION

In Maxwell's electromagnetic field theory, the field of a point-like charge is singular at the charge position and hence it has infinite self-energy. To overcome this problem in classical electrodynamics, the nonlinear electromagnetic field has been proposed by Born and Infeld [1], with the main motivation, to resolve the self-energy problem by imposing a maximum strength of the electromagnetic field. In this theory, the electric field of a point charge is regular at the origin and this nonlinear theory for the electromagnetic field was able to tone down the infinite self-energy of the point-like charged particle. Later, Hoffmann [2] coupled general relativity with Born–Infeld electrodynamics to obtain a spherically symmetric solution for the gravitational field of an electrically charged object.

Born-Infeld electrodynamics was first introduced in the 1930s to obtain a finite energy density model for the electron [1]. It has attracted considerable interest in recent times due to various reasons. One of the motivations being the observations that it arises naturally in open superstrings and in D-branes [2]. The low energy effective action for an open superstring in loop calculations leads to Born-Infeld type actions [3]. It has also been observed that the Born-Infeld action arises as an effective action governing the dynamics of vector-fields on D-branes [4]. For a review of aspects of Born-Infeld theory in string theory see Gibbons [5].

In recent times, the cosmological constant has been considered for several theoretical and observational reasons. One of the strongest support has been the recent results from a supernova which suggests a positive cosmological constant [6]. From the theoretical point of view, recent advances in string theory have been subjected to studies in all dimensions with a negative cosmological constant [7]. Asymptotically anti-de Sitter black holes are more interesting than their counterparts since they allow more geometries for black hole horizons than for the $\Lambda = 0$ case [8]. Also,

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they are thermodynamically stable, which has been an inspiration for much recent work [9]. Therefore, studying black holes in (anti)de Sitter spaces has an important place in current research.

II. BLACK HOLES IN BORN-INFELT GRAVITY

The action of the gravitational field coupled to a nonlinear Born–Infeld electrodynamics (or an Einstein–Born–Infeld action) in (3+1) dimensions reads [1, 2]

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{16\pi G}+\mathcal{L}(\mathcal{F})\right], \tag{1}$$

where R is scalar curvature, $g=det|g_{\mu\nu}|$, and $\mathcal{L}(\mathcal{F})$ is given by

$$\mathcal{L}(\mathcal{F})=\frac{\beta^2}{4\pi G}\left(1-\sqrt{1+\frac{2\mathcal{F}}{\beta^2}}\right), \tag{2}$$

with $\mathcal{F}=\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$; $F_{\mu\nu}$ denotes the electromagnetic field tensor. Here, β^2 is the Born-Infeld parameter, equal to the maximum value of electromagnetic field intensity, and it has the dimension of $[\text{length}]^{-2}$. Eq.(1) leads to the Einstein field equations,

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\kappa T_{\mu\nu}, \tag{3}$$

and the electromagnetic field equation

$$\nabla_\mu(F^{\mu\nu}\mathcal{L}_{,\mathcal{F}})=0. \tag{4}$$

The energy-momentum tensor is

$$T_{\mu\nu}=\mathcal{L}g_{\mu\nu}-F_{\mu\sigma}F^{\sigma}_{\nu}, \tag{5}$$

where $\mathcal{L}_{,F}$ denotes the partial derivative of $\mathcal{L}$ with respect to F.

The gravitational field of a static and spherically symmetric compact object with mass M and a nonlinear electromagnetic source in the Einstein-Born-Infeld theory has first been investigated by Hoffmann [2] and the space-time metric is [2, 50-53]

$$\mathrm{d}s^2=-\left[1-\frac{2GM}{r}+\frac{Q^2\left(r\right)}{r^2}\right]\mathrm{d}t^2+\left[1-\frac{2GM}{r}+\frac{Q^2\left(r\right)}{r^2}\right]^{-1}\mathrm{d}r^2+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\ \mathrm{d}\varphi^2\right), \tag{6}$$

with the square of the electric charge

$$Q^2(r)=\frac{2\beta^2r^4}{3}\left(1-\sqrt{1+\frac{Q^2}{\beta^2r^4}}\right)+\frac{4Q^2}{3}{}_2F^1\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{Q^2}{\beta^2r^4}\right), \tag{7}$$

where ${}_2F^1$ is the Gauss hypergeometric function.

Figure 1 demonstrates the radial dependence of the metric function and the Born-Infeld charge in the left and right panels, respectively. It is seen from the figure that the radius of event horizon radius slightly increases with the increase. However, the Born-Infelt charge decreases with the decrease of β in the region $r<M$, while at $r<M$ regions the charge increases.

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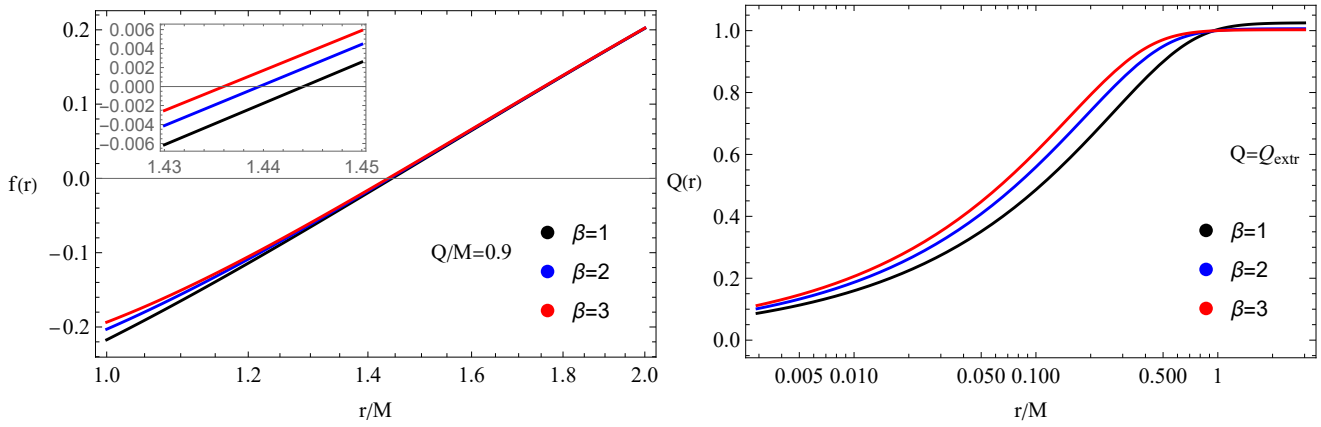


FIG. 1. Radial profiles of the metric function (left panel) and the Born-Infelt black hole charge (right panel) for the different values of the parameter β.

A. Scalar invariants

To explore of the properties of the spacetime may be started with the study of the scalar invariants of a geometry around a compact gravitating object. Here we analyse the Ricci scalar, Square of Ricci tensor and Kretchmann scalar of the spacetime metric (6) given with the lapse function given by Eq. (6).

Ricci scalar: Ricci scalar, so-called the scalar curvature, is one of the simplest curvature invariants of curved spacetime and defined as $R = g^{\mu\nu}R_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor. The positive and the negative values of the Ricci scalar correspond to sunken and convex form of the spacetime, respectively. After some simple mathematics, one may easily get the following form for the Ricci scalar,

$$R = \frac{4\beta^2\left(\sqrt{1+\frac{Q^2}{\beta^2 r^4}}-1\right)^2}{\sqrt{\frac{Q^2}{\beta^2 r^4}+1}}\,. \tag{8}$$

Square of Ricci tensor: Consider the second scalar invariant: so-called square of Ricci tensor, being responsible to the square of the energy momentum tensor of a field in the spacetime of a BH, and it defines as $\mathcal{R} = R_{\mu\nu}R^{\mu\nu} \equiv 1/(8\pi G)T_{\mu\nu}T^{\mu\nu}$ for the spacetime around the black holes (6) with the lapse functions given by Eq. (6). The square of Ricci tensor takes the following form

$$\mathcal{R} = \frac{8\beta^4\left(\sqrt{1+\frac{Q^2}{\beta^2 r^4}}-1\right)^2\left(Q^2+2\beta^2 r^4\right)}{Q^2+\beta^2 r^4}\,. \tag{9}$$

Kretchmann scalar: Now we consider the Kretchmann scalar defined as $\mathcal{K} = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$. Usually, the square root of the Kretschmann scalar can be interpreted as an effective gravitational energy density $\sqrt{\mathcal{K}} \sim \rho_M$. One can easily calculate the Kretchmann scalar for the spacetime metric (6) in the form

$$\begin{aligned} K = \frac{16}{3r^8\left(Q^2+\beta^2 r^4\right)}\Bigg\{ & \left(4Q^2\left(Q^2+\beta^2 r^4\right)\Big[-3Mr\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right)+Q^2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right)^2\right. \\ & \left.+Q^2\,{}_2F_1\left(\frac{3}{4},1;\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right)\Big]+9M^2r^2\left(Q^2+\beta^2 r^4\right)-6\beta^2 MQ^2r^5\sqrt{1+\frac{Q^2}{\beta^2 r^4}}+3\beta^2Q^4r^4 \right. \\ & \left.-4\beta^6r^{12}\left(\sqrt{1+\frac{Q^2}{\beta^2 r^4}}-1\right)-2\beta^4Q^2r^8\left(\sqrt{1+\frac{Q^2}{\beta^2 r^4}}-2\right)\right\}. \end{aligned} \tag{10}$$

It is seen from the expressions of the scalar invariants the invariants turns to the Schwarzschild limit when $\beta = 0$.

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III. DYNAMICS OF TEST PARTICLES

Here, we provide a brief overview of the studies regarding the motion of electrically neutral test particles around static BHs using the following Lagrangian:

$$L_{\rm p} = \frac{1}{2} m g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} \ , \tag{11}$$

with the constants of the motion

$$g_{tt}\dot{t} = -\mathcal{E} \ , \qquad g_{\phi\phi}\dot{\phi} = \mathcal{L}, \tag{12}$$

where $\mathcal{E} = E/m$ and $\mathcal{L} = L/m$ represent specific energy and angular momentum of the particle, respectively. The equations of motion for the particles are governed using so-called normalization condition:

$$g_{\mu\nu} u^{\mu} u^{\nu} = \epsilon \tag{13}$$

with ϵ taking value 0 for massless particles and -1 for massive ones.

For test particles, the equations of motion for time-like geodesics can be obtained using the condition given in Eq. (13), together with Eq. (12) in the following form :

$$\dot{r}^2 = \mathcal{E}^2 + g_{tt}\left(1 + \frac{\mathcal{K}}{r^2}\right) \ , \tag{14}$$

$$\dot{\theta}^2 = \frac{1}{g_{\theta\theta}^2}\left(\mathcal{K} - \frac{\mathcal{L}^2}{\sin^2\theta}\right) \ , \tag{15}$$

$$\dot{\phi} = \frac{\mathcal{L}}{g_{\phi\phi}} \ , \tag{16}$$

$$\dot{t} = -\frac{\mathcal{E}}{g_{tt}} \ , \tag{17}$$

where $\mathcal{K}$ is the constant for the angular motion also known as the Carter constant, and corresponds to the total angular momentum of the test particle.

Here, we assume that the motion of the particles occur in the equatorial plane where $\theta = \pi/2$, $\dot{\theta} = 0$ and the Carter constant is $\mathcal{K} = \mathcal{L}^2$, the equation for the radial motion of the particles becomes

$$\dot{r}^2 = \mathcal{E}^2 - V_{\rm eff} \ , \tag{18}$$

where the effective potential for the radial motion of test particles reads

$$V_{\rm eff} = f(r)\left(1 + \frac{\mathcal{L}^2}{r^2}\right) \ . \tag{19}$$

In order to compare the effects of charges of above-mentioned charged BHs, we consider extremely charged BHs. To study circular motion of the particles, we use the following conditions:

$$V_{\rm eff} = \mathcal{E}, \qquad V'_{\rm eff} = 0 \ . \tag{20}$$

In order to find the values of angular momentum and energy of particles in circular orbits, we used the conditions given in Eq.(20), to obtain:

$$\mathcal{L}^2 = \frac{r^2\left(-2Q^2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right) + 3Mr - 2\beta^2 r^4\left(\sqrt{\frac{Q^2}{\beta^2 r^4}+1}-1\right)\right)}{6Q^2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right) + 3r(r-3M)} \ , \tag{21}$$

$$\mathcal{E}^2 = \frac{\left(r\left(6M + 2\beta^2 r^3\left(\sqrt{\frac{Q^2}{\beta^2 r^4}+1}-1\right) - 3r\right) - 4Q^2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right)\right)^2}{9r^2\left(2Q^2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{Q^2}{r^4\beta^2}\right) + r(r-3M)\right)} \ . \tag{22}$$

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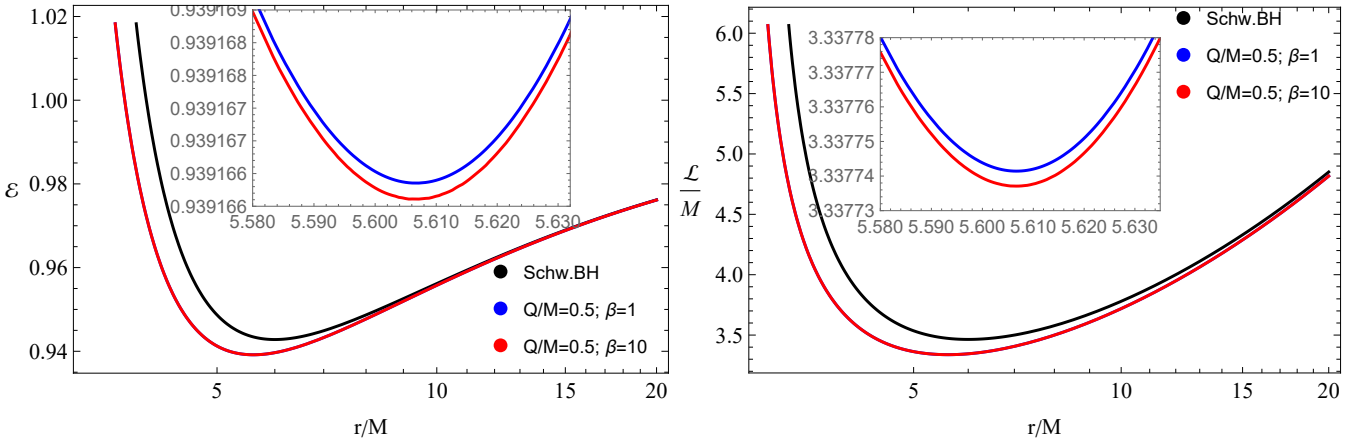


FIG. 2. Radial profiles of specific energy (left panel) and the specific angular momentum (right panel) of test particles in circular orbits for the characteristic values of BH charge Q.

The stability condition of circular orbits of test particles around a central point-like gravitating compact objects is described by the equation $\partial_{rr} V_{\text{eff}} = 0$. In most cases, this condition is applied together with the circularity condition given in Eq.(20), called the equation for ISCO.

Figure 23 gives information about specific energy and angular momentum at ISCO points and the particles only take these values while traveling around BH at ISCO region. The results are compared with Schwarzschild BH and the values of specific energy and specific angular momentum slightly vary for the different values of Born-Infeld parameter β when $Q = 0.5$. One can see from plots that the increased parameter β causes to rise of energy and angular momentum.

$$
\begin{aligned}
&-\frac{2Q^2}{\beta^2 r^3}\left(12M + 10\beta^2 r^3 - r\right)\left(Q^2 + \beta^2 r^4\right)\,{}_2F_1\left(\frac{3}{4}, 1; \frac{5}{4}; -\frac{Q^2}{r^4\beta^2}\right) \\
&+8Q^4\sqrt{\frac{Q^2}{\beta^2 r^4} + 1}\,{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{r^4\beta^2}\right)^2 + 4\left(3Q^4 + 5\beta^2 Q^2 r^4\right)\,{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{r^4\beta^2}\right) \\
&+r\left[18M^2 r\sqrt{\frac{Q^2}{\beta^2 r^4} + 1} - 3M\left(-10\beta^2 r^4\left(\sqrt{\frac{Q^2}{\beta^2 r^4} + 1} - 1\right) + r^2\sqrt{\frac{Q^2}{\beta^2 r^4} + 1} + 6Q^2\right)\right. \\
&\left.-8\beta^2 r^5\left(\sqrt{\frac{Q^2}{\beta^2 r^4} + 1} - 1\right) + 2Q^2 r\left(2\beta^2 r^2\left(\sqrt{\frac{Q^2}{\beta^2 r^4} + 1} - 1\right) + 1\right)\right] = 0\,,
\end{aligned}
\tag{23}
$$

Figure 3 show the dependence of ISCO radius from the black hole charge for the different values of Born-Infeld parameter. One can observe from the figure that an increase of the black hole charge decreases sufficiently the ISCO radius, while with the increase of the β parameter very slightly decreases it.

IV. SUMMARY AND DISCUSSIONS

In the present work, we study circular motion of test particles around charged black holes in Born-Infeld gravity. First, we have investigated radial dependence of the Born-Infeld black hole charge. It is found that the increase of the Born-Infeld parameter the charge increase at $r < M$ region, while out of $r = M$ region it slightly increases. Then, we have considered scalar invariants of the spacetime around Born-Infeld black holes such as Ricci scalar, square of Ricci tensor and Kretchmann scalar that are turns to the Schwarzschild limit when $\beta = 0$. We have analysed the effects of the black hole charge and the Born-Infeld parameter on the energy and angular momentum of the particles corresponding to circular motion of the particles. Finally, we have also investigated behaviour of the radius of innermost stable circular orbits of test particles with respect to the variation of the black hole charge for the different values of the Born-Infeld parameter. It is obtained that the Born-Infeld parameter almost does not change the ISCO radius.

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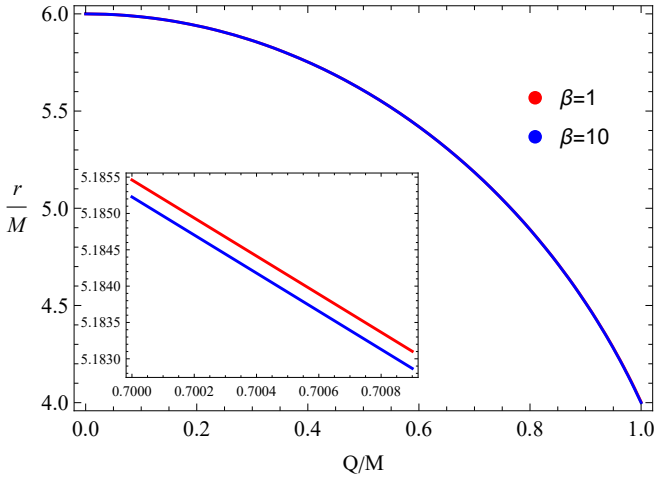


FIG. 3. The radius of ISCO versus the Born-Infelt black hole charge for the different values of the parameter β.

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