Journal of Fundamental and Applied Research **Vol. 2**, Issue 2 (2022) 20220011

Received: 12 May 2022 / Accepted: 31 May 2022
Published online: 30 June 2022
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Gravitational capture cross section of magnetized particles around Schwarzschild black hole in a uniform magnetic field

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Here we investigate the capture cross section of a magnetized particle orbiting around a black hole in external uniform magnetic field. The influence of magnetic parameter on capture cross of the radial motion of magnetized particle around the Schwarzschild black hole is studied. For studying the capture cross we use Hamilton–Jacobi formalism. It is found that circular orbits for photons and slowly moving particles may become stable near $r = 3M$. It was showed that the capture cross section of the magnetized particle is sensitive on the change of magnetic parameter.

I. INTRODUCTION

The actual moment of the amount of motion is proportional to intrinsic magnetic moment of a measurable particle value. With the spin of a particle and its direction in space the concept of the polarization of particles and the ensemble of particles, which can also be measured. The polarization of photons [1, 2], for example, is measured with using various polarizing devices, the simplest of which are optical prisms, and for measuring the polarization of electrons the method of intra-beam scattering, the annihilation method, the method of comton scattering, etc. Dynamics of the relative a particle that has an internal moment of momentum, vo external gravitational field was first considered in 1937 outstanding Polish scientist Miron Matisson [3, 4], who worked in Kazan University at the end of the 30s. A in 1951a. A. Papapetru, using the Fock method to derive the equations of motion, showed that a relative particle with an internal momentum does not move along geodesic, and as a result, re-discovered the Matisson equations, which since then have gained greater prominence under the name "Papapetru's equation", although the latter is often referred to "Mathisson-Papapetru" equations. It should be noted that the concept of "multipole particles", a similar concept of multipole moments, was introduced earlier by V.Tuluchev. The concept of multi-pole moments are used, for example, to study the motion of trials charged particles with spin in an external Resner-Nordstrem field and when discussing the motion of extended objects in a gravitational field created by the same multi-field objects. I.A. Khriplovich gave one of the most simple and general derivations of the equations of motion relative particle with an internal moment of quantity motion in an external gravitational field. He showed that an additional force that makes the motion of a particle non-Godesic, is the gravitational analogue of the Lorentz force. Contribution to study of spin dynamics in the framework of the general theory of relativity French mathematician N.-M.Suryu. He deduced the equations of dynamics particles with spin in external fields based on the variational principle. In contrast to the Matisson-Papetru equations, initially obtained in terms of the velocity vector, N.-M. equations in terms of the vector of the generalized momentum, which is in the in the case of multipole particles, the velocity vector is not linear. A The last time a wide interest in the study of dynamics particles with spin in a gravitational field, studying the interaction magnetized particles moving in a uniform magnetic field black holes. Along with studies of the motion of particles with spin

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in within the framework of the classical theory of gravity, in a number of works by A.A. Sokolov and D.V.Galutsov considered the radiation of relativistic particles in gravitational fields. First properties of an external magnetic field around black holes were studied by the American astrophysicist Robert Adolescence in the seventies of the last century [5]. We considered black a hole placed in an external asymptotically homogeneous magnetic field created by being near a source, such as a neutron star or a magnetar, an exact solution has been obtained Macwell's vacuum equations for this magnetic field. This was a brilliant idea, since it was shown by Ginsburg and Ozerny in 1964 [6], black holes do not have their own magnetic field. After This property of black holes in an external magnetic field has been studied in detail studied by various authors with the aim of studying the issue of extraction The energy of a black hole, in particular through the Blundford-Znack effect. Of particular interest is the movement of electrically charged, magnetic or neutral particles around the black hole, located in an external asymptotically uniform magnetic field. Worth noting is the contribution of the work of these movements to uniform, dipole magnetic and electric fields. In his works describing the interaction between the spinal part and the external magnetic and electrical fields. In particular, in the motion of magnetized part is considered in relation to the interaction between the external field and magnetic moment of the particles through β parameter which is the function of the radius of the vector, energy and angular momentum of the particles and of the intrinsic magnetic moment.

Here we consider capture cross section of magnetized particle around non-rotating Schwarzschild black hole and motion of magnetized particle in external magnetic field [7]. Here, capture cross section have been considered taking account of magnetic coupling parameter β [8, 9]. Following the idea of the paper [8, 10], we assume that the black hole is immersed in the external uniform magnetic field. We plan to study the influence of magnetic parameter and magnetic field on capture cross section of magnetized particle around compact object in external magnetic field. We obtain the equation of motion of the particles using Hamilton-Jacobi formalism [11].

II. FIELD AROUND THE BLACK HOLE

The main goal is to find the effect of the magnetic moment part of the black hole capture cross section. Here we display the analytical expressions for the cross section of the capture of magnetized particles. And also we study the direction of the uncaptured parts, that is, in which in the direction of the particle can break away from the capture of the black hole.

$$ds^2 = -A^2dt^2 + H^2dr^2 + r^2d\theta^2 + r^2\sin^2\varphi, \tag{1}$$

where the metric function

$$A^2 = H^{-2} = \left(1 - \frac{2M}{r}\right)$$

expressed in terms of the M parameter, which is the total mass black hole placed in an asymptotically homogeneous magnetic field B_0. For simplicity, the direction of the polar axis is chosen as as before, along the direction of the magnetic field.

The Hamilton-Cacobi equation for magnetized particles is chosen takim, in progress [8],

$$H = \frac{1}{2}\left(\frac{1}{m+U}g^{\mu\nu}P_\mu P_\nu + U\right), \tag{2}$$

from which one can easily derive the radial equation of motion magnetized test particles, as shown in:

$$r^3\left(\frac{dr}{d\tau}\right)^2 = f(r)\ , \tag{3}$$

where

$$f(r) = (e^2 + \eta - 1)r^3 + 2M(1-\eta)r^2 - l^2r + 2Ml^2\ , \tag{4}$$

e and l are, respectively, the energy and momentum of the particle, divided by the mass m and introduce the notation, as done in the work [8]

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TABLE I:

β	0	$1\cdot10^{-6}$	$5\cdot10^{-5}$	$5\cdot10^{-4}$	$1\cdot10^{-4}$	$5\cdot10^{-3}$	$1\cdot10^{-2}$
$r_{\rm ISCO}/M$	6	5.99	5.95	5.64	5.50	4.94	4.34
$l_{\rm cr}/M$	4	3.95	3.91	3.85	3.80	3.73	3.69

$$\eta = \frac{\beta\left(1-\frac{2M}{r}\right)}{\left(1-\frac{2M}{r}-\Omega^2 r^2\right)^{\frac{1}{2}}} \ . \tag{5}$$

The discriminant polynomial $f(r)$ in the expression (4) has The following view:

$$D = \left(\frac{l}{M}\right)^2 + \left(16+\frac{1}{\eta}\right)\left(\frac{l}{M}\right) - 16\left(\frac{1}{\eta}-3\right) \ , \tag{6}$$

where η is given by:

$$\eta = \frac{\beta\left(1-\frac{2M}{r}\right)}{\left(1-\frac{2M}{r}-\Omega^2 r^2\right)^{\frac{1}{2}}} \ . \tag{7}$$

Here we have neglected the η members of a higher order, proceeding from from the condition of its smallness.

Multipole solutions exist if the discriminant is multiple vanishes, i.e. $D = 0$. Then the exact solution for critical angular momentum of magnetized particles has the following view:

$$\frac{l_{\rm cr}}{M} = \sqrt{16+\frac{24}{\eta}+\frac{1}{4\eta^2}} - 8 - \frac{1}{2\eta} \ . \tag{8}$$

To get accurate values for critical angular momentum momentum of magnetized particles, the numerical method solves the equation for $l_{\rm cr}$. Numerical results for $l_{\rm cr}$ are presented in the II table. Based on the results presented in table, it is possible to conclude that in the presence of a magnetic interaction with an external magnetic field, the value of the critical the angular momentum of the magnetized particles decreases. This fact indicates that magnetized particles can come close to a black hole, in contrast to spinless particles. In the limiting case, t.e. when $\beta = 0$ the value of the critical angular momentum is equal to $l_{\rm cr} = 4$, which coincides with the critical the value of magnetized particles in the Schwartzchild space-time [12].

Fig 1 shows the dependence of the critical value momentum of the particles from the magnetic parameter β. Ec The figure shows that $l_{\rm cr}$ decreases with increasing β parameter. This means that the particle capture cross section decreases if the particle has a magnetic moment.

Here we consider the angle of direction of the particle, that is, in which direction, the particle must move so that it frees itself from capturing a black hole. We can write an equation of motion (6), for ultra-relative ($E \gg m$) particles with magnetic moment μ in metric (1) as follows:

$$\left(\frac{L^2}{r^2}\frac{dr}{d\phi}\right)^2 + V_{eff} = E^2, \tag{9}$$

Here V_{eff} is the effective potential of the particle and it has the form

$$V = \left(1-\frac{2M}{r}\right)\left(\frac{L^2}{r^2}-\eta\right). \tag{10}$$

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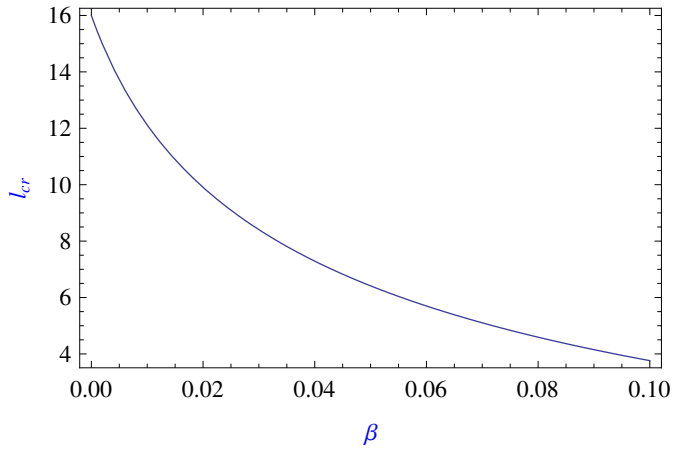


FIG. 1:

where, for a rough observer, you can write $\Omega = 0$, consequently we get

$$\eta = \beta\sqrt{1 - 2M/r}. \tag{11}$$

After elementary calculations we can get following expressions

$$\left(\frac{1}{r^2}\frac{dr}{d\phi}\right)^2 + \left(1 - \frac{2M}{r}\right)\left[\frac{1}{r^2} - \tilde{\beta}\left(1 - \frac{2M}{r}\right)^{\frac{1}{2}}\right] = \frac{1}{b^2}, \tag{12}$$

and

$$\frac{1}{b_{cr}^2} = \frac{1}{27M^2} - \frac{\tilde{\beta}}{3}\left(1 - \frac{2M}{r}\right)^{\frac{1}{2}}. \tag{13}$$

A close observer in Schwarzschild coordinates can calculate particle velocity in the following form [12]:

$$\vartheta_r = \pm\sqrt{1 - b^2/B^2} \qquad \vartheta_\phi = b/B. \tag{14}$$

Where $B = \sqrt{(1 - 2M/r)(1/r^2 - \eta)}$ and $\theta_{\rm cr} = \arccos\vartheta_r = \arcsin\vartheta_\phi$.

We can find the angle $\theta_{\rm cr}$, between propagation direction and radial particle direction:

$$\theta_{\rm cr} = \arcsin\left[3M\sqrt{3\left(1 - \frac{2M}{r}\right)\left(\frac{1}{r^2} - \tilde{\beta}\left(1 - \frac{2M}{r}\right)^{\frac{1}{2}}\right)}\right]. \tag{15}$$

The physical meaning of the critical angle $\theta_{\rm cr}$ is possible explain using the diagram in fig.2. Particles with an angle less than $\theta_{\rm cr}$ captured by the central black hole, in the opposite case, the particle can break away from a black hole.

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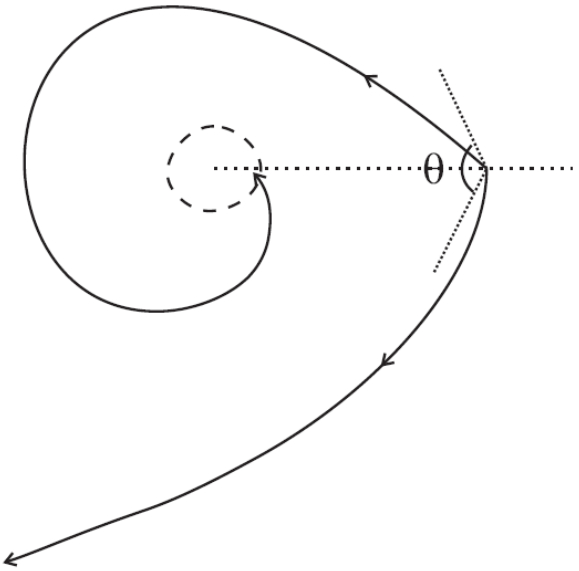


FIG. 2: Trajectories of particles. If the initial the direction of motion of the particle is less than θ, the particle can life is captured by the central black hole, otherwise particles can break away from a black hole. dashed line represents the events of the black hole horizon.

III. SUMMARY AND CONCLUSION

Of greatest interest is the search for the critical value angular moment of magnetized particles. Particles with a moment, the value of which is less critical hole. It is well known that for neutral particles the critical the value of the angular momentum is equal to $L_{\rm cr} = 4M$. In this In our work, we have shown that in the presence of an external magnetic field and of the magnetic moment of the particle critical value of the angular momentum decreases, t. e. magnetized particles can avoid capture black hole in the presence of the β parameter.

We also considered the direction of motion of a particle, which can freeing itself from the capture of a black hole, i.e. the angle is determined, necessary for the release of a part of the black hole. We discovered Radial dependence of the critical angle $\theta_{\rm cr}$ particles for different values of the β parameter. Particles directed with an angle whose value is lower than $\theta_{\rm cr}$, in the end The count will fall into the black hole. We have come to the conclusion that in the presence magnetic parameter β, the critical value of the angular moment decreases. Particles with a magnetic moment are more stable in comparison with the classical particles around the black holes in a Schwarzschild immersed in an external magnetic field. At the end We have found the radial dependence of the critical angle $\theta_{\rm cr}$ for partials at different values of the parameter β.

Acknowledgements

The research is supported in part by Grants F-FA-2021-432 and MRB-2021-527 of the Uzbekistan Ministry for Innovative Development.

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