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Electronic specific-heat of high-T_c cuprates in the superconducting state

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In this work, we study the temperature dependence of the electronic specific heat $C_e(T)$ of high-T_c cuprates in the superconducting state at low temperatures and near the critical temperature T_c of the superconducting transition. We show that the low-temperature specific heat $C_e(T)$ of these high-T_c superconductors predicted by the theory of three-dimensional superfluid Bose-liquid exhibits the phonon-like $C_e(T) \sim T^3$ behavior. We identify the nature of jump-like anomalies in $C_e(T)$ near the superconducting transition temperature T_c. In this case the total electronic specific heat $C_e(T)$ of high-T_c cuprates below T_c is calculated taking into account different contributions coming from the excited components of Cooper pairs, the ideal Bose-gas of incoherent Cooper pairs, the unpaired carriers in the impurity band and superfluid bosonic Cooper pairs. We have shown that our theoretical predictions of the behaviors of $C_e(T)$ at $T << T_c$ and near T_c are strikingly similar to the behaviors of the electronic specific heat observed at low temperatures and near T_c in $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{7-d}$. There is a fair quantitative agreement between theoretical predictions of the behaviors of $C_e(T)$ and experimental data.

Introduction

The electronic properties of high-T_c cuprate superconductors are fundamentally different from those of conventional superconductors. The normal and superconducting properties of underdoped and optimally doped cuprates are unusual in respect to the behaviors of conventional superconductors [1-5]. The important superconducting properties of high-T_c cuprates closely related to the mechanism of superconductivity are their thermal properties, which are significantly different from those of conventional superconductors [5-10]. Measurements of the electronic specific heat $C_e(T)$ in $La_{2-x}Sr_xCuO_4$ (LSCO) and $YBa_2Cu_3O_{7-d}$ (YBCO) showed a clear λ-like anomaly near the superconducting transition temperature T_c [6,7,10] and a linear term at low temperatures [11]. It seems more likely that the linear term in the specific heat of cuprate superconductors is not intrinsic property of the superconducting state in high-T_c cuprates, but due to the presence of some impurity phases [12]. It is often misinterpreted that the λ-like anomaly in $C_e(T)$ at T_c is the BCS-type jump of the specific heat. Actually, a λ-like superconducting transition in cuprate superconductors occurring at T_c is quite different from a BCS-type transition corresponding to the formation of Cooper pairs. So far, the origins of the electronic specific-heat properties of high-T_c cuprates at low temperatures and near T_c have not been fully understood. Many experiments indicate (see Refs. [13-15]) that unconventional electron-phonon interactions (including polaronic effects) may be involved in the high-T_c superconductivity in the cuprates.

In this work, we study the specific heat properties of high-T_c cuprates in the superconducting state at low temperatures and near T_c. We argue that the relevant charge carriers in doped cuprates are large polarons residing in the polaronic band and localized hole carriers residing in the impurity band. The electronic specific heat $C_e(T)$ of the cuprate superconductors LSCO and YBCO below T_c is calculated by considering different contributions coming from the excited components of polaronic Cooper pairs, the ideal Bose-gas of incoherent (non-superconducting) Cooper

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pairs, the unpaired carriers in the impurity band and the superfluid bosonic carriers (polaronic Cooper pairs). The total electronic specific heat $C_e(T)$ below T_c is calculated by considering the contribution $C_s(T)$ coming from the superfluid bosonic carriers exhibits the phonon-like ($C_s(T) \sim T^3$) behavior at low temperatures ($T << T_c$) and λ-like anomaly near T_c. Numerical results obtained for $C_e(T)$ at $T << T_c$ and near T_c are compared with experimental data. For all cases considered, good quantitative agreement is found between theory and experiment.

Possible contributions to $C_e(T)$ coming from different types of charge carriers in doped high-T_c cuprates

It is natural to assume that different types of charge carriers can coexist in doped high-T_c cuprates below T_c and contribute to $C_e(T)$ [16,17]. Such charge carriers are (i) the holes or electrons localized on the impurities and residing in the impurity band, (ii) the excited components of polaronic (bosonic) Cooper pairs, (iii) the incoherent (non-superconducting) bosonic Cooper pairs, and (iv) the superconducting bosonic carriers (i.e. polaronic Cooper pairs). Possible contributions of the three types of non-superconducting charge carriers to $C_e(T)$ were considered previously [16,17]. Here we examine the possible behaviors of $C_e(T)$ in different high-T_c cuprates taking into account these three contributions and the contribution of the specific heat $C_s(T)$ coming from the superfluid bosonic Cooper pairs at low temperature ($T << T_c$) and near T_c. In so doing, we determine the contribution of $C_s(T)$ to the total electronic specific heat $C_e(T)$ from the relation [18,19]

$$C_s(T) = \frac{1}{k_B T^2} \sum_k n_B(k)(1 + n_B(k)) E_B^2(k) - T[\varepsilon \frac{\partial \tilde{\mu}_B}{\partial T} + \frac{1}{2} \frac{\partial}{\partial T} \Delta_g^2], \quad (1)$$

where $n_B(k) = [\exp(E_B(k)/k_BT) - 1]^{-1}$ is the Bose distribution function, $E_B(k) = \sqrt{\tilde{\varepsilon}_B^2(k) - \Delta_B^2}$ is the excitation spectrum of interacting bosons, $\tilde{\varepsilon}_B(k) = \varepsilon(k) + \tilde{\mu}_B(k)$ is the energy of these bosons measured from the renormalized chemical potential $\tilde{\mu}_B$, $\varepsilon(k) = \hbar^2 k^2/2m_B$, $\Delta_g = \sqrt{\tilde{\mu}_B^2 - \Delta_B^2}$ is the energy gap in the excitation spectrum $E_B(k)$ of a superfluid Bose-liquid, Δ_B is the coherence parameter (i.e. superfluid or superconducting order parameter) of condensed bosons, $\vec{k}$ is the wave vector of bosons, m_B is the mass of these bosons.

Electronic specific heat of high-T_c cuprate superconductors at low temperatures $T << T_c$

At low temperatures $T << T_c$, one can assume that $\tilde{\mu}_B = const$, $\Delta_B = const$ and the specific heat of a three-dimensional (3D) superfluid Bose-liquid is given by [19]

$$C_s(T) = \frac{\Omega D_B}{4k_B T^2} \int_0^\infty \sqrt{\varepsilon} \frac{E_B^2(\varepsilon)}{\sinh^2[E_B(\varepsilon)/2k_BT]} d\varepsilon, \quad (2)$$

where Ω is the volume of the system, $D_B = m_B^{3/2}/\sqrt{2}\pi^2\hbar^3$ is the density of states of bosons.

When the interboson coupling constant $\gamma_B = \tilde{V}_B D_B \sqrt{\xi_{BA}}$ (where $\tilde{V}_B$ is the potential of the effective attractive interaction between bosons, ξ_{BA} is the cutoff energy for the attractive part of the interboson interaction potential $V_B(k)$) smaller than some critical value $\gamma_B = \gamma_B^*$, the energy gap in the excitation spectrum of a 3D superfluid Bose-liquid vanishes below a certain characteristic temperature $T_c^* < T_c$ [19]. Therefore, at $\gamma_B \leq \gamma_B^*$ and $T \leq T_c^*$ the excitation spectrum of a 3D superfluid Bose-liquid becomes gapless and phonon-like

$$E_B(\varepsilon) \sim \sqrt{\frac{|\tilde{\mu}_B|}{m_B}} \hbar k. \quad (3)$$

In this case the specific heat of a 3D superfluid Bose-liquid is expected to be proportional to T^3 at low temperatures $T << T_c$, i.e.,

$$C_s(T) \sim T^3. \quad (4)$$

Actually, experimental results indicate that the power low temperature dependence of the electronic specific heat of high-T_c cuprates, $C_e(T) \sim T^3$, is observed at low temperatures [20]. It follows that at $T << T_c$ the main contribution to $C_e(T)$ gives the specific heat $C_s(T)$ of a superfluid Bose-liquid of Cooper pairs.

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Electronic specific heat of high-T_c cuprates near superconducting transition temperature T_c

In high-T_c cuprates the polaronic carriers are bound into bosonic Cooper pairs above T_c and a small part of these Cooper pairs condense into a Bose-liquid state at T_c. Since the concentration ρ_B of superfluid bosonic Cooper pairs is turned out to be much more smaller than the concentration n_B of preformed bosonic Cooper pairs. The superconducting order parameter Δ_B appearing below T_c [18,19] is differs from the BCS-like gap appearing above T_c as a pseudogap. The occurrence of superconductivity in the bosonic cuprate superconductors is tied to the coherence parameter $\Delta_B = \Delta_{SC}$, which defines bond strength of all condensed bosons. Where as the BCS-like gap defines the bond strength of the Fermi components of Cooper pairs which exist above T_c. For a superfluid Bose-liquid of Cooper pairs, the chemical potential $\tilde{\mu}_B(\geqslant \Delta_B)$ and the coherence parameter $\Delta_B = \Delta_{SC}$ near T_c are given by $\tilde{\mu}_B(T) \simeq \tilde{\mu}_B(T_c)[1 + a(T_c - T)^{0.5}]$ and $\Delta_B(T) \simeq 2\tilde{\mu}_B(T_c)\sqrt{a}(T_c - T)^{0.25}$ [19], where $a = 2(\xi_{BA}/k_BT_c)^{0.25}/(c_0\gamma_BT_c)^{0.5}$ and $c_0 = \pi^{3/2}/3.912$. We now turn to the temperature dependence of the specific heat of this Bose superfluid, which diverges as $C_s(T) \sim (T_c - T)^{-0.5}$ near T_c (where $\Delta_B(T) << \tilde{\mu}_B(T) << k_BT_c$) [13] and will exhibit a λ-like anomaly at T_c, (i.e., the behavior of $C_s(T)$ is similar to that of the specific heat of superfluid ^{4}He). The temperature derivatives of $\tilde{\mu}_B$ and Δ_B entering the expression for $C_s(T)$ give rise to such a λ-like divergence near T_c (see Fig. 1). By introducing the quantity of superfluid matter $\nu_B = N_B/N_A$ (where N_B is the number of attracting bosons (polaronic Cooper pairs) and N_A is the Avogadro number, which is equal to the number of CuO_2 formula unit per unit molar volume) and the molar fraction of the superfluid bosonic carriers defined by $f_s = \nu_B/\nu$ (where $\nu = N/N_A$ is the amount of doped matter, N is the number of doped charge carriers), we can now write the molar specific heat of the superconducting Bose-liquid in high-T_c cuprates as

$$C_{sm}(T) = f_s\frac{C_s(T)}{\nu_B} = f_s\frac{D_BN_A}{4\rho_Bk_BT^2} \times \\ \times \int_0^{\varepsilon_{BA}} \frac{\sqrt{\varepsilon}d\varepsilon}{\sinh^2\frac{E_B(\varepsilon)}{k_BT}}\left\{E_B^2(\varepsilon) + \frac{a\tilde{\mu}_B(T_c)T}{2(T_c - T)^{0.5}}\left[\varepsilon - \tilde{\mu}_B(T_c)\right]\right\}. \quad (5)$$

Here we have accounted for that $\Omega/\nu_B = N_Bv_B/\nu_B = v_BN_A$ and $v_B = 1/\rho_B$. As mentioned above, the doped carriers in the cuprates are distributed between the polaronic band (with the Fermi energy ε_F) and the impurity band (with the Fermi energy ε_{FI}) and the specific heat $C_n(T)$ of non-superconducting carriers below T_c is calculated by considering three contributions from the excited components of polaronic Cooper pairs, the ideal Bose-gas of incoherent Cooper pairs and the unpaired carriers bound to impurities [16]. In this case, the fraction f_p of doping carriers residing in the polaronic band and the other fraction f_I of such carriers residing in the impurity band should be taken into account in comparing the specific heat $C_s(T)$ of the superconducting Bose-liquid of Cooper pairs with experiment. Thus, the total specific heat $C_e(T) = C_s(T) + C_n(T)$ of superconducting and non-superconducting carriers below T_c is calculated and compared with the experimental data for $C_e(T)$ in high-T_c cuprates (Fig. 2).

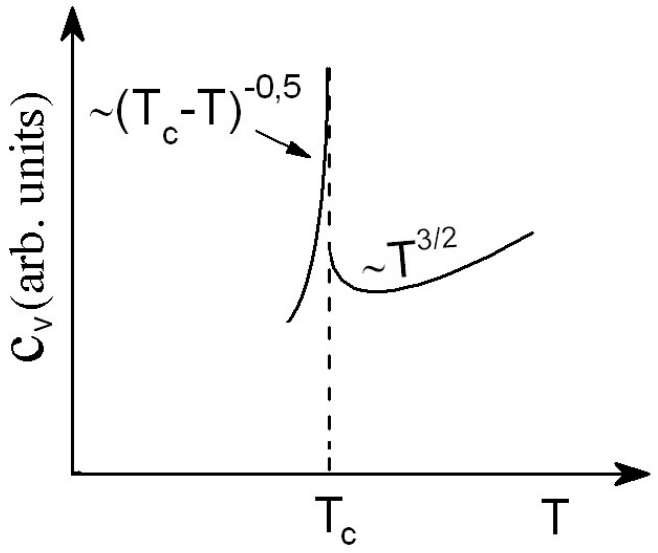


FIG. 1: The specific heat of a superfluid 3D Bose-liquid exhibiting a λ-like anomaly near T_c.

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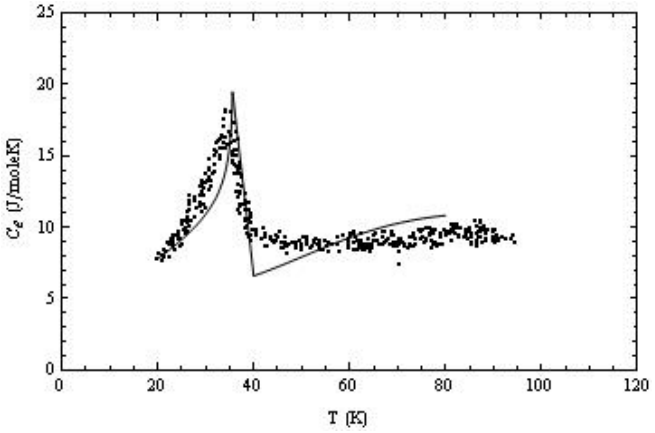


FIG. 2: Temperature dependence of the electronic specific heat of LSCO measured near T_c and above T_c [21]. The solid line is the calculated curve for comparing with experimental points (black circles). According to [16], the contribution of $C_n(T)$ to $C_e(T)$ is calculated by using the parameters $\varepsilon_F = 0.1$ eV, $\varepsilon_{FI} = 0.06$ eV, $f_p = 0.4$, $f_I = 0.6$, while the contribution of $C_s(T)$ to $C_e(T)$ is calculated by using the parameters $\rho_B = 1.4 \times 10^{19} cm^{-3}$, $m_B = 2.7 m_p$, $\tilde{\mu}_B(T_c) = 0.5$ meV and $f_s = 0.012$.

Conclusion

We have studied the electronic specific heat properties of high-T_c cuprates at low temperatures $T << T_c$ and in the vicinity of T_c. We have found that at $T << T_c$ the specific heat of a 3D superfluid Bose-liquid of Cooper pairs in these materials has the power low temperature dependence (i.e. $C_s(T) \sim T^3$), which is in agreement with the experimental observation. We have considered the high-T_c material below T_c as a multi-carrier system. The total electronic specific heat $C_e(T)$ in the vicinity of T_c is calculated by considering the contribution of $C_s(T)$ coming from the superfluid bosonic Cooper pairs and the contribution of $C_n(T)$ coming from the excited Fermi components of Cooper pairs, the ideal Bose-gas of incoherent Cooper pairs and the unpaired charge carriers in the impurity band. The calculated results for $C_e(T)$ are in fair quantitative agreement with the experimental data for $C_e(T)$ in high-T_c cuprates. We have shown that our theoretical predictions of the behaviors of $C_e(T)$ at low temperatures $T << T_c$ and near T_c are strikingly similar to the behaviors of the electronic specific heat properties observed in these high-T_c materials. Our calculated results for $C_e(T)$ in the vicinity of T_c also closely resembles the observed λ-like behavior of $C_e(T)$ near T_c.

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