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Radiation and test particles around $4D$ Einstein-Gauss-Bonnet black holes

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In the present work, we have studied thermodynamics of Schwarzschild black holes in novel 4D Einstein Gauss Bonnet (4 D EGB) theory and Hawking radiation around the spacetime. We have investigated studied the motion of test particles around the black hole. It is obtained that both the specific energy and angular momentum of the test particles decrease with the increase of the GB parameter α. We also studied the effects of the GB parameter on radius of event horizon, photonsphere (last circular orbits for photons) and innermost stable circular orbits (ISCOs). Finally, the binding energy of the test particle orbiting around the black hole in 4D EGB theory is also investigated. It is obtained that the energy efficiency decreases for negative value of α parameter, while it increases at positive values of parameter α.

I. INTRODUCTION

In general relativity (GR) black holes (BHs) have been known as a generic result of Einstein gravity as their geometric properties are described by simple mathematical equations. It is worth to note that the recent experiments and astrophysical observations related to the gravitational waves [1, 2] and the first image of the supermassive BH shadows [3, 4] can allow understanding the nature of the geometry and test the different gravity models in the strong field regime. Although, Einstein theory has the limits of applicability where it can lose its predictive power. Thus, for its validity and applicability, higher order theories are proposed for possible extensions of GR [5]. In this context, the Lovelock theory can be considered as a generalization of Einstein's theory [6] as it is a higher order theory which exist in D dimensions. Gauss-Bonnet (GB)/Lovelock gravity with the quadratic order gives contribution to the equation of motion only in $D > 4$. However, it was recently proposed that the novel Einstein-Gauss-Bonnet (EGB) theory could exist even in D = 4 dimensions, thus allowing to avoid the Lovelock's theorem by rescaling the GB coupling constant [7]. However, the resulting theory seems to have more degrees of freedom than Gauss-Bonnet gravity in 4D and is argued to be a scalar-tensor theory. The problem was revisited by Aoki, Gorji and Mukhohyama (AGM) and they obtained a consistent theory of 4D Gauss-Bonnet gravity by dimensionally reducing $D = d + 1$ theory. The theory breaks temporal diffeomorphism invariance, but still preserves spatial diffeomorphism invariance in the limit $d \to 3$.Throughout this work we use signature $(-, +, +, +)$ for the space-time and geometrized unit system $G = c = 1$ (However, for an astrophysical application we have written the speed of light explicitly in our expressions). Latin indices run from 1 to 3 and Greek ones from 0 to 3.

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II. SPACETIME METRIC FOR $4D$ EGB BH

The metric of the spacetime around Schwarschild BH in novel 4D is given by Gauss-Bonnet gravity is given by,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \ , \tag{1}$$

where the lapse function $f(r)$ is defined as

$$f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha M}{r^3}}\right) \ , \tag{2}$$

where M is the total mass of BH and α is GB coupling parameter with range $-8 \leq \alpha/M^2 \leq 1$. In the absence of the α parameter, the BH solution leads to get Schwarzschild solution in GR:

$$\lim_{\alpha\to 0} f(r) = 1 - \frac{2M}{r} \ . \tag{3}$$

Meanwhile, the asymptotic behavior of the lapse function in Eq.(2) is given by

$$f(r) \simeq 1 - \frac{2M}{r} + \frac{4\alpha M}{r^3} + \mathcal{O}(r^{-5}) \ . \tag{4}$$

Despite the complicated form of the lapse function the radii of horizon, which is determined from the condition $f(r) = 0$, takes a form

$$r_\pm = M\left(1 \pm \sqrt{1 - \frac{\alpha}{M^2}}\right) \ , \tag{5}$$

which is very similar to the horizon radii in RN or Kerr spacetime. Once we find horizon radii then one can immediately calculate the entropy, which is proportional to surface area of the black hole $A = 4\pi r_+^2$, by

$$S = \frac{A}{4} = \pi r_+^2 = \pi M^2\left(1 + \sqrt{1 - \frac{\alpha}{M^2}}\right)^2 \ , \tag{6}$$

which reduces to $S = 4\pi M^2$ in Schwarzschild space. The Hawking temperature for $4D$ EGB black hole is calculated by,

$$T = \frac{1}{4\pi}\frac{r_+^2 - \alpha}{r_+(r_+^2 + 2\alpha)} \ , \tag{7}$$

which tends to $T = 1/(8\pi M)$ in Schwarzschild space (i.e. $\alpha = 0$). Another important thermodynamic quantity of a system is the free energy, $F = M - ST$ which takes a form

$$F = M - \frac{r_+}{4}\left(\frac{r_+^2 - \alpha}{r_+^2 + 2\alpha}\right) \ , \tag{8}$$

The black hole surface (left panel), the temperature of Hawking radiation around the black hole (middle panel), and the free energy in spacetime (black hole) termodynamics (right panel) as a function of the GB coupling parameter α are shown in Fig.1. It is seen from the figure the black hole surface and Hawking temperature decrease with respect to the decrease of α, while the free energy increase.

In an inelastic process, the specific heat of the thermodynamic system can be found as

$$\begin{aligned} C_V &= T\frac{\partial S}{\partial T} = T\frac{\partial S}{\partial M}\left(\frac{\partial T}{\partial M}\right)^{-1} \\ &= \pi M^2\frac{r_+^2 - \alpha}{r_+\left(2\alpha + r_+^2\right)} \ , \end{aligned} \tag{9}$$

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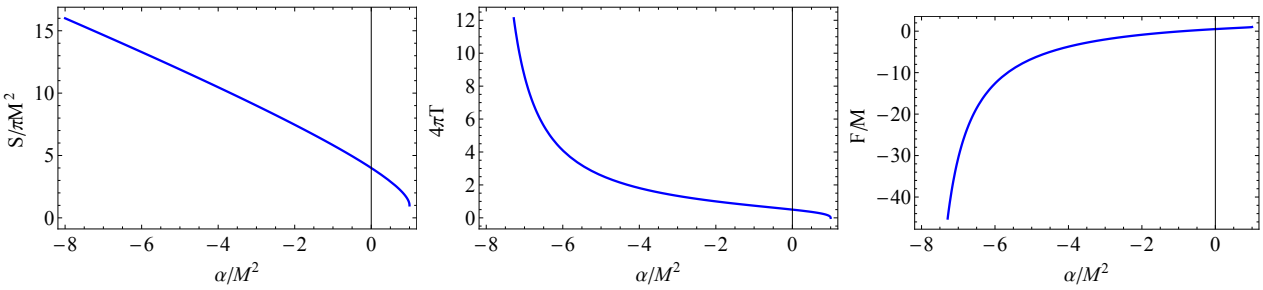


FIG. 1: The dependence of the black hole surface (left panel), the temperature of Hawking radiation around the black hole (middle panel), and the free energy in spacetime (black hole) termodynamics (right panel) on the GB coupling parameter α.

The most important parameter of BH is its lifetime. The rate of energy loss from BH in the $4D$ EGB gravity can be approximated with the Stephan-Boltzmann radiation law as,

$$\frac{dM}{dt} \simeq \sigma A T^4 = 4\pi\sigma r_+^2 T^4 \ , \tag{10}$$

where σ is the Stephan-Boltzmann constant. Using the expressions for the temperature (7) and radius of horizon (5) one can rewrite equation (10) in the form

$$\frac{dM}{dt} = 4\pi\sigma \frac{\left(r_+^2 - \alpha\right)^4}{r_+^2\left(2\alpha + r_+^2\right)^4} \ . \tag{11}$$

III. PARTICLE MOTION

Since the spacetime metric (1) is independent of coordinates t and ϕ, then there should exist conserved quantities p_t and p_ϕ, which are energy and angular momentum of the test particle at infinity. It is well known that the four-velocity of test particle, i.e. $u^\alpha = \dot{x}^\alpha = dx^\alpha/d\lambda$, where λ is an affine parameter, is satisfied the following normalization condition: $g_{\alpha\beta}u^\alpha u^\beta = -1$ or more precisely

$$\dot{r}^2 + f(r)r^2\dot{\theta}^2 = \mathcal{E}^2 - V_{\text{eff}}(r,\theta) \ , \tag{12}$$

where the effective potential $V_{\text{eff}}(r,\theta)$ is defined as

$$V_{\text{eff}}(r,\theta) = f(r)\left(1 + \frac{\mathcal{L}^2}{r^2\sin^2\theta}\right) \ , \tag{13}$$

here $\mathcal{E}$ and $\mathcal{L}$ are, respectively, the specific energy and the specific angular momentum of test particle at infinity.

Standard analysis shows that particle motion is limited by the energetic boundary conditions given by $\dot{r} = 0$ and $\dot{\theta} = 0$ which leads

$$V_{\text{eff}}(r,\theta) = 0 \ . \tag{14}$$

On the other hand the stationary points of the effective potential (13) can be found from the following conditions

$$\partial_r V_{\text{eff}}(r,\theta) = 0 \ , \qquad \partial_\theta V_{\text{eff}}(r,\theta) = 0 \ . \tag{15}$$

allows finding extremes (r_0,θ_0) of the function. From the second expression in (15) one can easily find that one of the extreme point of the effective potential is located at $\theta_0 = \pi/2$ which corresponds to an equatorial plane and another extreme point r_0 of the effective potential can be found from the first expression in (15). Using the Eq.(14) and the first expression in (15) the radial dependence of the specific energy and the specific angular momentum at circular orbit can be found as

$$\mathcal{E}^2 = f(r)\left(1 + \frac{r^2\left(1 + \frac{2\alpha M}{r^3} - \zeta\right)}{2\alpha\left(\frac{3M}{r} - \zeta\right)}\right) , \tag{16}$$

$$\mathcal{L}^2 = \frac{r^4\left(1 + \frac{2\alpha M}{r^3} - \zeta\right)}{2\alpha\left(\frac{3M}{r} - \zeta\right)} , \tag{17}$$

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where ζ is defined as

$$\zeta = \sqrt{1 + \frac{8\alpha M}{r^3}} \ . \tag{18}$$

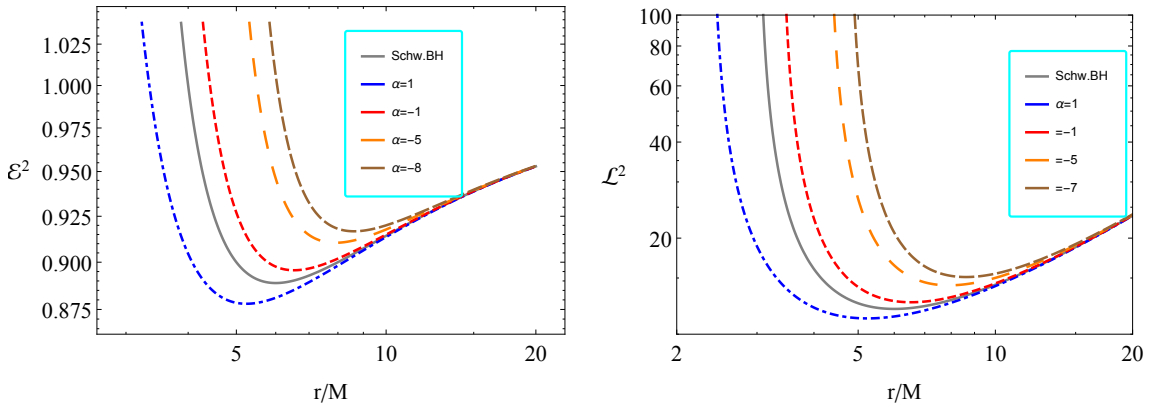


FIG. 2: The radial dependence of specific energy and angular momentum of the test particle for circular orbits for the different values of the GB parameter α.

In Fig.2, we present radial dependencies of the specific energy and angular momentum of the test particles around a Schwarzschild black hole in 4D EGB gravity. It is observed from the figure that both quantities, the specific energy and angular momentum, are decreased with the increase of α.

Another important quantity is the angular velocity measured by a distant observer, i.e. $\Omega = d\phi/dt$, which reads

$$\Omega = \Omega_K \sqrt{\frac{r^3}{2\alpha M} - \frac{1}{\zeta}\left(1 + \frac{r^3}{2\alpha M}\right)} \ , \tag{19}$$

where $\Omega_K = \sqrt{M/r^3}$ is the well-known Keplerian angular frequency. One can easily check that absence of α parameter the angular velocity in Eq.(19) gives exactly the Keplerean frequency ($\Omega = \Omega_K$).

From now, we focus on the characteristic radii of test particle around $4D$ EGB BH. Since the photon is massless, its specific energy and the specific angular momentum should tend to infinity, which means the denominator of expressions (16) and (17) should be zero. The radius of photonsphere is determined as solution of the following cubic equation $r^3 - 9M^2 r + 8\alpha M = 0$, which takes a form

$$r_{\rm ph} = 2\sqrt{3}M \cos\left[\frac{1}{3}\arccos\left(-\frac{4\alpha}{3\sqrt{3}M^2}\right)\right] \ . \tag{20}$$

There exists another radii of massive particle so-called innermost stable circular orbit (ISCO) which can be found the following condition $\partial_r^2 V_{\rm eff}(r,\theta) = 0$ along the conditions (15), which allows writing the equation:

$$\frac{\zeta}{\alpha}\left[4 + \frac{15M}{r} + \frac{4\alpha M}{r^3}\left(8 + \frac{3M}{r}\right)\right]$$
$$-\frac{40\alpha M^2}{r^6} - \frac{2M}{r^3}\left(23 + \frac{42M}{r}\right) - \frac{1}{\alpha}\left(\frac{15M}{r} + 4\right) = 0 \ . \tag{21}$$

The solution of Eq.(21) presents ISCO radius (i.e. $r_{\rm ISCO}$) for test particle, and it is difficult to find analytical solution for it. Absence of α parameter one can obtain general relativistic limit for ISCO radii i.e. $r_{\rm ISCO} = 6M$.

Figure 3 presents characteristic radii, such as radius of event horizon, photonsphere (last circular orbits for photons) and ISCO for test particle orbiting around $4D$ EGB BH. We have shown that the presence of the negative values of the GB parameter cause to increase the characteristic radii, while positive values cause to decrease them.

The binding energy of the test particle orbiting around the black hole is determined by the difference of its rest energy and the energy given at certain orbit, i.e. $E_{\rm bind} = (1 - \mathcal{E}_{\rm ISCO})mc^2$, which allows writing an expression for the energy efficiency for the test particle as

$$\eta = 1 - \mathcal{E}_{\rm ISCO} \ , \tag{22}$$

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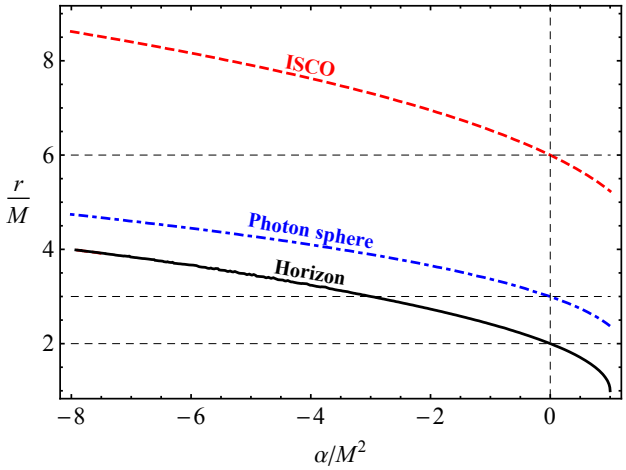


FIG. 3: Dependence of the characteristic radii, horizon $r_{\rm h}$, photonphere $r_{\rm ph}$, and ISCO $r_{\rm ISCO}$, from the α parameter.

where $\mathcal{E}_{\rm ISCO}$ is the specific energy of the test particle at ISCO. In order to find it, we simply calculate the value of the specific energy (16) at ISCO radii.

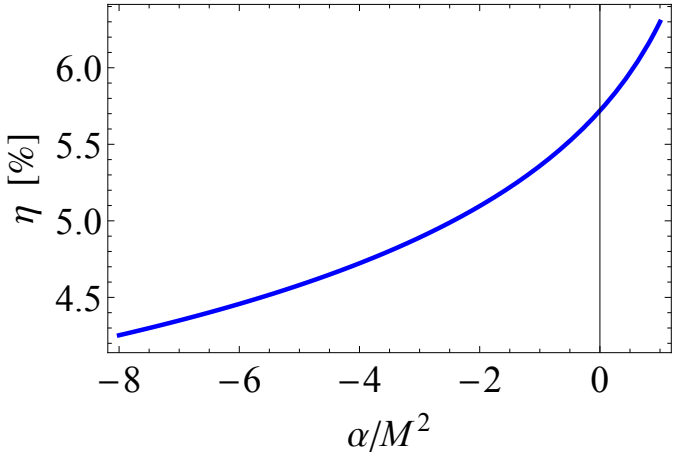


FIG. 4: Dependence of the energy efficiency from the α parameter.

Figure 4 draws dependence of the energy efficiency from α the parameter. It can be easily seen that the energy efficiency decreases for negative value of α parameter, while it increases at positive values of parameter α.

IV. SUMMARY AND DISCUSSIONS

In the present work, we have studied thermodynamics of Schwarzschild black holes in novel 4D Einstein Gauss Bonnet and Hawking radiation around the spacetime. It is obtained that the black hole surface and Hawking temperature decrease with respect to the decrease of α, while the free energy increase. We have also studied the motion of test particles around the black hole. We have shown that both the specific energy and angular momentum of the test particles decrease with the increase of the GB parameter α. Moreover, our studies on the effects of the GB parameter on radius of event horizon, photonsphere (last circular orbits for photons) and ISCO have shown that the presence of the negative values of the GB parameter cause to increase the characteristic radii, while positive values cause to decrease the radii. Finally, The binding energy of the test particle orbiting around the black hole in 4D EGB theory is also investigated. It is obtained that the energy efficiency decreases for negative value of α parameter, while it increases at positive values of parameter α.

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