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Test particle motion around regular black holes

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In this work, we study four potential of electromagnetic field around a regular Ayon-Beato-Garcia black hole (ABG BH), together with analysis of energy and effective potential for the circular motion of test particles around the BH. We also investigate the effective potential for circular motion of photons around the ABG BH, taking account the interaction between photons and non-linear electrodynamics field of the BH.

I. INTRODUCTION

The black holes governed by standard general relativity contain a physical singularity with diverging Riemann tensor components and predictability breakdown, considered as a realm of quantum gravity overcoming this internal defect of general relativity. However, families of regular black hole solutions of Einstein's gravity have been found that eliminate the physical singularity from the spacetimes having an event horizon. Of course, these are not vacuum solutions of the Einstein equations, but contain necessarily a properly chosen additional field, or modified gravity, so that the energy conditions related to the existence of physical singularities are then violated. In looking at these, we used the unit system $G = c = 1$. In the below, we will focus on the main properties of the ABG black hole.
The line element of regular ABG black holes is given as, [1]

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + d\Omega^2 \quad ; \quad f(r) = 1 - \frac{2Mr^2}{(r^2+Q^2)^{3/2}} + \frac{Q^2r^2}{(Q^2+r^2)^2} \tag{1}$$

the associated electric field

$$E_r(r) = Qr^4\Big[\frac{r^2-5Q^2}{(r^2+Q^2)^4} + \frac{15}{2}\frac{M}{(r^2+Q^2)^{7/2}}\Big] \tag{2}$$

One can find

$$A_t = \int E_r dr = \frac{Qr^5}{2}\left[\frac{3M}{Q^2\left(Q^2+r^2\right)^{5/2}} - \frac{2}{\left(Q^2+r^2\right)^3}\right] + C$$

here C is integral constant. One can find the constant using limit when $q \to 0$ scalar electric potential also should be zero.

$$\lim_{q\to 0}\frac{Qr^5}{2}\left(\frac{3M}{Q^2\left(Q^2+r^2\right)^{5/2}} - \frac{2}{\left(Q^2+r^2\right)^3}\right) + C = \frac{3M}{2Q} + C$$

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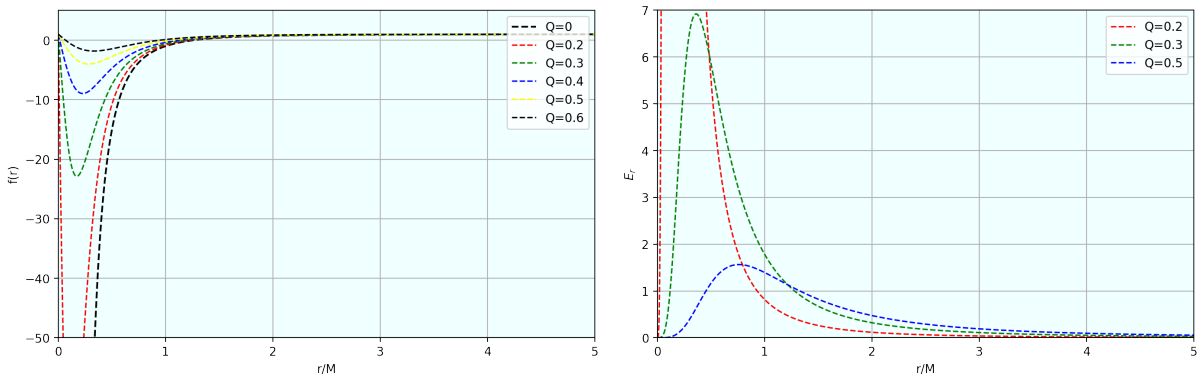


FIG. 1: Radial dependence of f(r) (on the left) and E_r on the right.

then $C = -\frac{3M}{2Q}$

In the follow given the radial dependence of the $f(r)$ function and electric field (see Fig.1).

In Fig.1, we demonstrate the lapse function of the ABG BH (left panel) and the electric field (right panel) as function of the radial coordinate for various values of the BH charge Q. It is observed from the left panel that the radius of outer (inner) horizon decrease (increase) with respect to the increase of Q. One can see from the right panel that the electric field increase with the charge decreasing.

Event horizon From $f(r) = 0$ we can calculate event horizon radius for ABG black hole:

$$1 - \frac{2Mr^2}{(r^2+Q^2)^{3/2}} + \frac{Q^2r^2}{(Q^2+r^2)^2} = 0 \Rightarrow r^4 + Q^4 + 3r^2Q^2 - 2Mr^2\sqrt{r^2+Q^2} = 0 \tag{3}$$

If we conditionally choose M and Q in equation (3) as one unit, we can get the solution of equation (3) with respect to r. The analysis of the roots shows that the above expression constitutes positive real roots if $0 < Q < 2M$. The critical value of charge $Q_c = 0.63M$ separates the black hole and the no-horizon ABG spacetime. In the no-horizon case, the ABG spacetime is regular at all radii $r \geq 0$. For $Q \geq Q_c$, there are no horizons while the given spacetime admits two horizons for $Q < Q_c$ leading to a non-extremal black hole. The Schwarzschild black hole is recovered for $Q = 0$.

II. TEST PARTICLE MOTION AROUND ABG BLACK HOLE

Effective potential of neutral and charged test particles given in the follow 4th and 5th equations[1]:

$$V_{eff} = f(r) \cdot (1 + \frac{L^2}{r^2}) = \left(\frac{L^2}{r^2} + 1\right)\left(-\frac{2Mr}{(q^2+r^2)^2} + \frac{q^2r^2}{(q^2+r^2)^2} + 1\right) \tag{4}$$

From the Fig.2, we can see that the charge of the test particle does not change its effective potential much, while the charge of the black hole significantly changes the effective potential of the particle. In conclusion, we can say that the space-time properties around the black hole depend only on the parameters of the black hole.

III. PHOTON ORBITS

The photon motion in the spacetime of a Bardeen BH in 4-D EGB gravity has determined by the modified null geodesics of the effective geometry due to interaction between photons and NED field, as we mentioned above, defined

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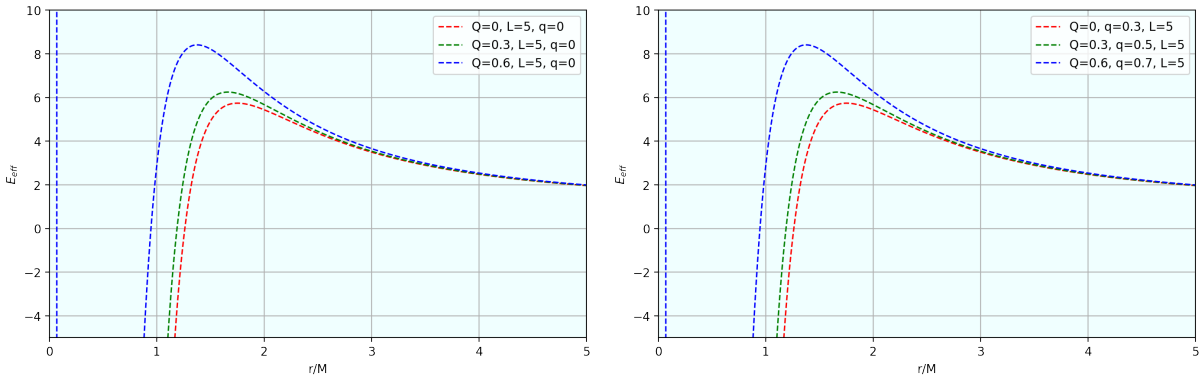


FIG. 2: Radial dependence of effective potential, neutral particle (left panel) and charged particle (right panel).

as [2]

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - 4\frac{L_{FF}}{L_F}F^{\lambda}_{\mu}F^{\mu\nu} , \tag{5}$$

$$\tilde{g}_{\mu\nu} = 16\frac{L_{\rm FF}F_{\mu\eta}F^{\eta}_{\nu} - (L_{\rm F} + 2FL_{\rm FF})g_{\mu\nu}}{F^2L^2_{\rm FF} - 16(L_{\rm F} + FL_{\rm FF})^2} , \tag{6}$$

$$\mathrm{d}\tilde{s}^2 = -\frac{f(r)}{L_F}\,\mathrm{d}t^2 + \frac{1}{L_F f(r)}\mathrm{d}r^2 + \frac{r^2}{\Phi}\mathrm{d}\theta^2 + \frac{r^2}{\Phi}\mathrm{d}\phi^2, \tag{7}$$

where

$$L_{\rm F} = \frac{\partial L(F)}{\partial F}, \quad L_{\rm FF} = \frac{\partial^2 L(F)}{\partial F^2} . \tag{8}$$

$$\Phi = L_F + 2L_{FF}F \tag{9}$$

The motion of photons in the spacetime around regular BHs in GR coupled to NED can be described in so-called, eikonal equation,

$$\tilde{g}_{\mu\nu}k^{\mu}k^{\nu} = 0 , \tag{10}$$

where k^{μ} is the four-wave vector related to the four-momentum of photons by the relation $p^{\mu} = k^{\mu}$ ($\hbar = 1$). The effective potential for the radial motion of photons around regular BH with the NED Lagrangian has the following form [3, 4]

$$V_{\rm eff}(r) = \frac{f(r)}{r^2}\left(1 + 2\frac{L_{\rm FF}}{L_{\rm F}}F\right) . \tag{11}$$

For LED ($L(F) = F$) the effective potential turns the standard form,

The local minimum of the radial profile of the effective potential corresponds to the photon circular orbits. We call b as impact parameter of a light that reaches infinity, it can be defined as follows

$$b \equiv \frac{\mathcal{L}}{\mathcal{E}}, \tag{12}$$

where both quantities $\mathcal{L}$ and $\mathcal{E}$ are conserved because the effective metric (7) does not dependent to time t and azimuthal angle ϕ. The radius of the photonsphere and impact parameter of circular orbits of photons can be found by setting the condition $V'_{\rm eff} = 0$, where the prime $'$ denotes the derivative with respect to the radial coordinate.

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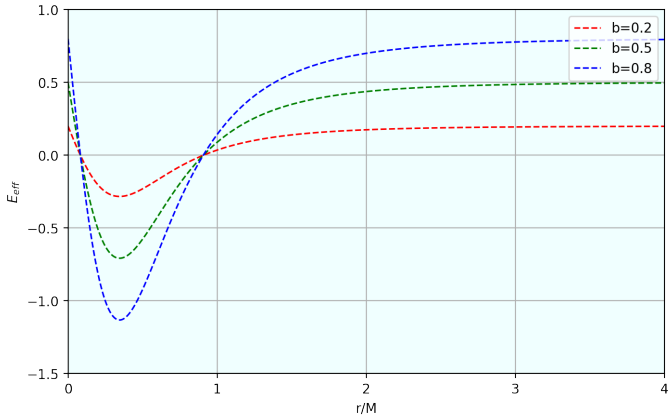


FIG. 3: Effective potential for photons around ABG BHs, with the extreme charge, different values of b.

IV. SUMMARY AND CONCLUSION

In this work, we have studied four potential of electromagnetic field around a regular Ayon-Beato-Garcia black hole (ABG BH), it is shown that the BH has two horizons (outer and inner) and the increase of the BH charge cause to come close each other by increasing inner horizon and decreasing the outer one. We have analysed the energy and effective potential for the circular motion of test particles around the BH. It is obtained that an increase of the BH charge leads to increase both energy and effective potential for circular motion. We also investigate the effective potential for circular motion of photons around the ABG BH, taking account the interaction between photons and non-linear electrodynamics field of the BH.

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