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Basic properties of nonlinearly charged rotating black hole

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The basic properties of the new type of solution has been investigated in the current work. Kretschmann scalar which is useful to investigate the singularity properties of black hole solutions has benn calculated and discussed. It has been shown the behavior of the metric component g^{rr} with the change of the radial coordinate r for selected values of spacetime parameters of the given spacetime. It has also been investigated the change of the event horizon radius with the change of the spacetime parameters mentioned. It has been used a system of units in which $G = c = 1$.

Introduction

In the work [1] it has been obtained a new type of black hole solution describing nonlinearly charged black hole with the spacetime metric

$$\begin{aligned} ds^2 &= -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 + \left(\Delta - a^2 - r^2\right)\frac{2a\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\ &+ \left(\frac{\left(a^2+r^2\right)^2\sin^2\theta}{\Sigma} - \frac{a^2\Delta\sin^4\theta}{\Sigma}\right)d\phi^2 \end{aligned} \tag{1}$$

with $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 + a^2 - 2Mr + q^2(1-\beta(r^2+a^2))^2$. Here, M defines the mass parameter, a corresponds to the spin parameter, q defines the electromagnetic charge (described as $q^2 = q_e^2 + q_m^2$ with q_e being electric charge and q_m being magnetic charge) and β is a coupling constant associated to the nonlinearity of the electrodynamics field potentials (see [2]). As can be seen the nonlinearity term affects the spacetime metric through the function Δ. It can be easily noticed that in the case when $\beta = 0$ the spacetime reduces to the well-known Kerr-Newmann solution and in the absence of electromagnetic charge the Kerr solution is recovered.

Let us now calculate Kretschmann scalar to investigate the singularity properties of the given spacetime. The Kretschmann scalar is defined as $\mathcal{K} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ which takes the following explicit form when $\theta = \pi/2$

$$\begin{aligned} \mathcal{K} &= \frac{8}{r^8}\Big[-4Mq^2r\left(3a^4\beta^2 + a^2\beta\left(\beta r^2 - 6\right) - \beta r^2 + 3\right) + q^4\left(7a^8\beta^4 + 2a^6\beta^3\left(\beta r^2 - 14\right) + 2a^4\beta^2\left(\beta^2 r^4 - 3\beta r^2 + 21\right)\right. \\ &\left. + 2a^2\beta\left(\beta^3 r^6 - 2\beta^2 r^4 + 3\beta r^2 - 14\right) + 3\beta^4 r^8 - 2\beta^3 r^6 + 2\beta^2 r^4 - 2\beta r^2 + 7\right) + 6M^2r^2\Big]. \end{aligned} \tag{2}$$

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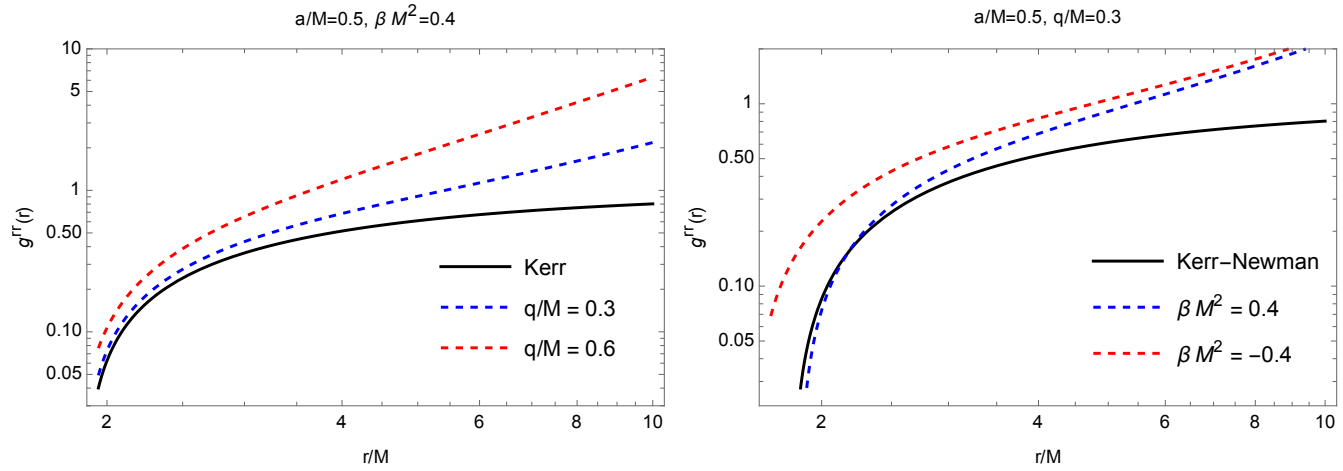


FIG. 1: The radial dependence of the metric component g^{rr} for the selected values of parameters q and β.

It can be easily seen that the spacetime has a physical singularity at $r \to 0$. It is also noticeable that in the asymptotic infinity the Kretschmann scalar does not tend to zero but equals to $\mathcal{K} = 24(\beta q)^4$. This in turn means that the given spacetime is not asymptotically flat in the presence of nonlinearity term which comes to the game trough β.

Before we start studying the event horizon properties let us see how the metric component g^{rr} changes with the change of radial coordinate r. In Fig. 1 it has been shown the radial dependence of g^{rr} for various values of spacetime parameters q and β. It can be seen that the given metric components tends to 1 only in the absence of β (the case $q = 0$ is not much interesting in this work). This is due to the fact that the given spacetime is not asymptotically flat. It can be also mentioned that the metric component takes bigger values for bigger q and smaller β.

One can now calculate the event horizon radius from the condition $g^{rr} = 0$. In Fig. 2 it is shown the dependence of the event horizon radius of a black hole described by the given spacetime for different values of spacetime parameters a, q, and β. From the top-left panel one can see that the event horizon radius grows up first, reaches the peak, and then starts going down with the grow of the parameter β. In the top-right panel it is shown that the event horizon radius decreases with the increase of the electromagnetic charge parameter q but the rate of change strongly depends on the sign of the coupling constant β. It is clearly seen that the change is relatively smaller for positive β and becomes considerable for zero or negative values of the latter. Lastly, in the bottom panel of Fig. 2 it is presented the change of the event horizon radius with the change of the spin parameter a for various values of β and fixed q. One can see that the event horizon radius monotonically decreases with the rise of the spin parameter and the rate of such change is dependent from the parameter β. It can be seen that positive values of β does not affect the event horizon radius much. However, negative values of this parameter changes the rate considerably.

Conclusion

In the current work we have considered nonlinearly charged rotating black hole solution which was recently obtained. We have studied some basic properties of the chosen spacetime. We have calculated the Kretschmann scalar to see if the given spacetime suffers from the singularity problem which is common for most black hole solutions. We have seen that it does. Moreover, it was calculated that the given spacetime is asymptotically non-flat which is interesting. Next, we have investigated the behavior of the metric component g^{rr} and found that this component also does not tend to the Minkowski one at asymptotic infinity. Finally, we have shown the dependence of the event horizon radius from the spacetime parameters. It has been shown that the dependence of the event horizon radius is not monotonic with the growth of β. It has been also demonstrated that the sign of this coupling constant plays an important role when investigating the change of the event horizon radius with the change of the rest two spacetime parameters, namely the spin parameter a and the electromagnetic charge parameter q. We have seen that negative values of β affects these changes considerably stronger than the positive values of this parameter.

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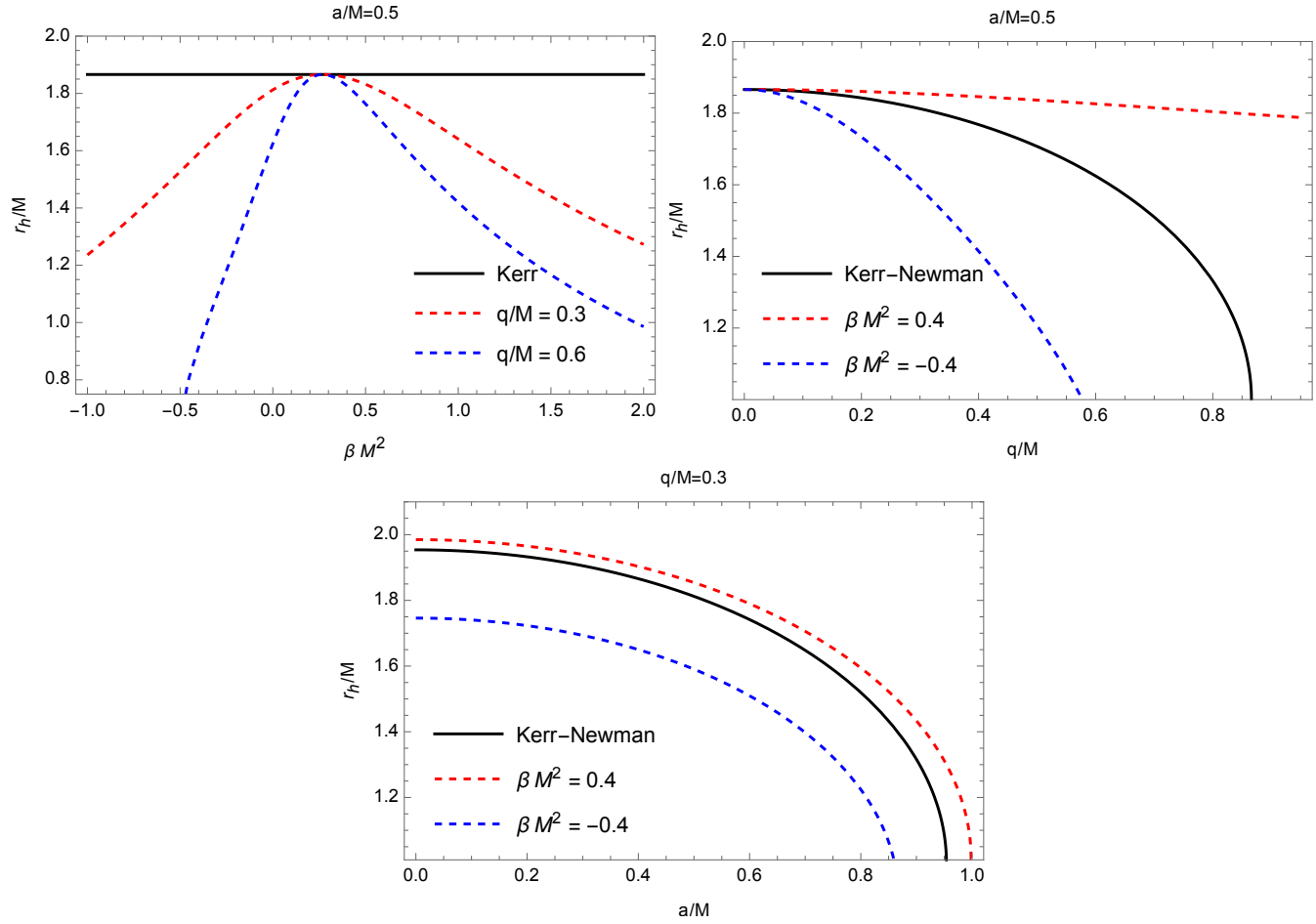


FIG. 2: The change of the event horizon radius with the change of the the nonlinearity coefficient β (top-left), electromagnetic charge q (top-right), and spin parameter (bottom).

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