Journal of Fundamental and Applied Research **Vol. 2**, Issue 1 (2022) 20220002

Received: 24 January 2022 / Accepted: 11 February 2022
Published online: 30 March 2022
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Spinning magnetized particles around magnetized Schwarzschild black hole

Uktambek Eshimbetov,[1,2,*] Temurmalik Tolibjonov,[3,†] Ziyodaxon Jalolova,[4,‡] and Javlon Rayimbaev[5,§]

[1] *National University of Uzbekistan, Tashkent 100174, Uzbekistan*
[2] *Institute of Nuclear Physics, Ulugbek 1, Tashkent 100214, Uzbekistan*
[3] *Namangan State University, Bobur shoh Str. 161, Namangan 160107, Uzbekistan*
[4] *Urgench State University, Kh.Alimdjan str 14, Urgench 220100, Uzbekistan*
[5] *Ulugh Beg Astronomical Institute, Astronomy St. 33, Tashkent 100052, Uzbekistan*

In this work, we derive effective potential for the radial motion of test spinning magnetized particles around Schwarzschild black holes immersed in an external uniform magnetic field using Mathisson-Papapetrou-Dixon equation. In the effective potential, we included in the term which responsible for the interaction between external magnetic field and dipole magnetic moment of the particles. In our future analyses, we aim to test the effects of the spinning and magnetic interaction parameters on the motion of rotating neutron stars around magnetized supermassive black holes.

I. INTRODUCTION

Even though most astrophysical bodies have spins and evolve in an expanding cosmological background, their motion is described well by ignoring the cosmic expansion and under the nonspinning test particle approximation for large distances from a central massive body and for relatively low spin values [1]. These approximations however become less accurate for large values of the spin and/or when the mass of the cosmic fluid inside the particle orbit becomes comparable to the mass of the central massive object. For such systems, new types of interactions appear which are proportional to the time derivatives of the cosmic scale factor and the spin of the orbiting particle. For example, phantom dark energy models can lead to dissociation of all bound systems in the context of a Big-Rip future singularity [2]-[4]. Also, the spin-curvature interaction can modify the motion of the test particles in black hole spacetimes [3] due to spin-spin or spin-orbit couplings, or make the motion chaotic thus modifying significantly the orbit of the test body leading to the emission of characteristic forms of gravitational waves.

II. TEST PARTICLE MOTION

The geometry of the spacetime around the BH at the center of the galaxy can be defined, in spherical polar coordinates, ($x^\alpha = \{t, r, \theta, \phi\}$) in the form of standard Schwarzschild metric:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{1}$$

with the metric function,

*eshimbetov1703@mail.ru
†moonlightwalkertemur@gmail.com
‡jalolovaziyoda@gmail.com
§javlon@astrin.uz

$$f(r) = 1 - \frac{2M}{r} \tag{2}$$

The Mathisson-Papapetrou-Dixon equation for the motion of spinning test particles

$$\frac{Dp^\mu}{d\tau} + \frac{1}{2} R^\mu_{\nu\alpha\beta} u^\nu S^{\alpha\beta} = 0 \tag{3}$$

$$\frac{DS^{\mu\nu}}{d\tau} - p^\mu u^\nu - p^\nu u^\mu = 0 \tag{4}$$

where, the covariant derivative reads as $D/d\tau \equiv u^\mu \nabla_\mu$ and $R^\mu_{\nu\alpha\beta}$ is the Riemann tensor.

On the other hands, Hamilton Jacobi equation for magnetized particles around black holes immersed in external magnetic fields,

$$p^\mu p_\nu = -m^2 \left(1 - \frac{1}{2m} D^{\mu\nu} F_{\mu\nu}\right)^2 \tag{5}$$

and the spin tensor of the particle

$$S^{\mu\nu} S_{\mu\nu} = 2S^2 = 2m^2 s^2 \tag{6}$$

where the product $D^{\mu\nu}F_{\mu\nu}$ corresponds to magnetic interaction of The polarization tensor for the spinning magnetized particle with zero electric charge is zero ($D^{\mu\nu} = (q/m)S^{\mu\nu} \equiv 0$), however, the tensor due to magnetic dipole moment of the particle defines by the following expression:

$$D^{\alpha\beta} = \eta^{\alpha\beta\sigma\nu} u_\sigma \mu_\nu, \qquad S^{\alpha\beta} = \eta^{\alpha\beta\sigma\nu} u_\sigma s_\nu \tag{7}$$

One can easily see from Eq.(7) that in the case when the particle's magnetic moment and spin axes of the particles is orthonormal to four-velocity of the particle (that means the magnetic dipole moment and spin has only θ component: $\mu^\alpha = (0, \mu^\theta, 0)$ and $s^\alpha = (0, s^\theta, 0)$) the condition satisfies,

$$S^{\alpha\beta} u_\beta = 0\ , \qquad D^{\alpha\beta} u_\beta = 0 \tag{8}$$

The effective mass of magnetized particles in an electromagnetic field is $m_{\text{eff}} = m - (1/2) D^{\alpha\beta} F_{\alpha\beta}$.

One can find the product $D^{\alpha\beta}F_{\alpha\beta}$ taking into account the condition given in Eqs. (7) and (8) in the following form,

$$D^{\mu\nu} F_{\mu\nu} = 2\mu^{\hat{\alpha}} B_{\hat{\alpha}} = 2\mu B_0 \sqrt{f(r)} \tag{9}$$

where $\mu^2 = \mu_\alpha \mu^\alpha$ is the norm of the dipole magnetic moment of magnetized particles. Here we have assumed that the direction of the magnetic dipole moment of the particles is perpendicular to the equatorial plane being parallel to the external magnetic field and

Considering the motion of the particle at the equatorial plane makes simplify our further calculations.

In fact, the magnetized particle's magnetic dipole moment has interaction with only external magnetic field that does not break the conservative quantities p_t and p_ϕ of spinning particles

$$\Sigma p_t = -E - \frac{s f'(r)}{2r} L \tag{10}$$

$$\Sigma p_\phi = L + sE \tag{11}$$

where $\Sigma = 1 - s^2 f'(r)/(2r)$

Similarly, the effective potential for radial motion of magnetized spinning particles defines as

$$V^{\pm}_{\text{eff}}(r; \beta, s, \mathcal{L}) = \frac{1}{\Delta} \left\{ s\mathcal{L} \left[2f(r) - r f'(r)\right] \pm \left[2r - s^2 f'(r)\right] \sqrt{f(r) \left[\mathcal{L}^2 + \Delta\left(1 - \beta\sqrt{f(r)}\right)\right]} \right\} \tag{12}$$

where $\Delta = 2(r^2 - s^2 f(r))$.

One can see from Eq. (12) that in order to have $(p_r)^2 \geq 0$, the energy of the particle must satisfy one of the following conditions: (i) $E < V_-$ or (ii) $E > V_+$. Hereafter, we focus on the case of spinning test particle with positive energy which coincides with the effective potential to be $V_{\text{eff}} = V_+$

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III. CONCLUSION

This work is devoted to derive the effective potential for the radial motion of test spinning particles with a magnetic dipole moment around uniformly magnetized Schwarzschild black holes. To do this, we use the Mathisson-Papapetrou-Dixon equation for the interaction betwen spactime and spin of particles by including in the term which responsible for the interaction between external magnetic field and dipole magnetic moment of the particles. In our future analyses, we plan to develop our studies in testing the effects of the spinning parameter together with magnetic interaction between external magnetic field and proper magnetic fields of the test particles on the motion of rotating neutron stars around magnetized supermassive black holes.

Acknowledgements

This research is supported by Grants F-FA-2021-432, F-FA-2021-510, and MRB-2021-527 of the Uzbekistan Ministry for Innovative Development.

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